COUPLING AND PROPAGATION OF SINGULARITIES IN THE INITIAL LAYER FOR BOLTZMANN EQUATION

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Abstract

In the study of the relationship between the Boltzmann equation in kinetic theory and fluid dynamics, it is essential to understand the singular layers. In the classical paper by Harold Grad, he identified three basic layers: boundary, shock, and initial layers. In this talk, I will present joint work with Tai-Ping Liu and Shih-Hsien Yu on the formation and propagation of singularities in the initial layer of the solutions to the Boltzmann equation in both the space-time (x, t) and microscopic velocity $\boldsymbol{\xi}$ domains. Singularities transport in the space-time (x, t) domain and interact with the nonlinear collision operator in the microscopic velocity $\boldsymbol{\xi}$ domain, creating rich singular coupling in the $(x, t, \boldsymbol{\xi})$ domain. Our work identifies the essential features of the collision operators using generalized Carleman-Hilbert coordinates, ensuring that the Green's function approach is sufficient to reveal the explicit structure of the singularities in the initial layers.