

# Local Exact Lagrangian Controllability For The 1D Compressible Navier–Stokes Equations

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## Abstract

We consider barotropic compressible Navier–Stokes equations in the interval  $[0, \pi]$  with homogeneous Dirichlet boundary conditions. Our result is the following: given two sufficiently close subintervals  $I$  and  $J$  of  $(0,1)$ , we construct a smooth external force  $f$  in the momentum equation supported in  $(1, \pi)$  such that the flow map moves  $I$  exactly onto  $J$  in a given time  $T > 0$ . The essential point in the proof is to find two external forces  $f_1$  and  $f_2$  that have "independent" stretching effect on  $I$ . Such forces are constructed using the linearized adjoint system and the independence is proved using a unique continuation property which we prove based on Fourier analytic techniques. This is a joint work with Franck Sueur (Université du Luxembourg) and Gastón Vergara-Hermosilla (Université Paris-Saclay).