Bulletin of the Institute of Mathematics
Academia Sinica (New Series)
Vol. **14** (2019), No. 3, pp. 385-388
DOI: 10.21915/BIMAS.2019304

ERRATUM TO "LEVEL-RANK DUALITY FOR VERTEX OPERATOR ALGEBRAS OF TYPES B AND D"

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1. Erratum

This is an erratum to the article "Level-rank duality for vertex operator algebras of types B and D" [3].

In [3], we determined the commutant vertex subalgebra $C_{L_{\widehat{\mathfrak{so}}_m}(1,0)^{\otimes n}}(L_{\widehat{\mathfrak{so}}_m}(n,0))$ (cf. Theorems 4.13, 4.15 and 4.17). While the main result holds for $m, n \geq 4$, it is not correct when $n = 3$. The correct statement should be amended as follows:

(1) For $m, n \geq 4$,

$$C_{L_{\widehat{\mathfrak{so}}_m}(1,0)^{\otimes n}}(L_{\widehat{\mathfrak{so}}_m}(n,0)) = L_{\widehat{\mathfrak{so}}_n}(m,0)^G$$

if m or n is odd and

$$C_{L_{\widehat{\mathfrak{so}}_m}(1,0)^{\otimes n}}(L_{\widehat{\mathfrak{so}}_m}(n,0)) = (L_{\widehat{\mathfrak{so}}_n}(m,0) \oplus L_{\widehat{\mathfrak{so}}_n}(m, m\Lambda_1))^G$$

if both m, n are even.

(2) For $m \geq 4$ and $n = 3$,

$$C_{L_{\widehat{\mathfrak{so}}_m}(1,0)^{\otimes 3}}(L_{\widehat{\mathfrak{so}}_m}(3,0)) = (L_{\widehat{\mathfrak{sl}_2}}(2m,0) + L_{\widehat{\mathfrak{sl}_2}}(2m,2m))^G$$

AMS Subject Classification: 17B69.
Key words and phrases: Level rank duality, vertex operator algebras, affine Lie algebras.

if m is even and

$$C_{L_{\widehat{\mathfrak{so}}_m}(1,0)^{\otimes 3}}(L_{\widehat{\mathfrak{so}}_m}(3,0)) = L_{\widehat{\mathfrak{sl}_2}}(2m,0)^G$$

if m is odd.

Here G is an abelian subgroup of Aut $(L_{\widehat{so}_{mn}}(1,0))$ such that $L_{\widehat{so}_{mn}}(1,0)^G$ is isomorphic to the tensor product VOA $L_{\widehat{so}_m}(1,0)^{\otimes n}$ (see [3, p.44] for detail).

1.1. Fermionic vertex superalgebras

As in [3], we consider the Clifford algebra $\mathcal{C}l_{2m}$ generated by $\psi_i^{\pm}(r), 1 \leq i \leq m$, $r \in \mathbb{Z}+\frac{1}{2}$, satisfying the non-trivial relations

$$[\psi_i^{\pm}(r), \psi_j^{\mp}(s)]_+ = \psi_i^{\pm}(r)\psi_j^{\mp}(s) + \psi_j^{\mp}(s)\psi_i^{\pm}(r) = \delta_{r+s,0}\delta_{ij},$$

where $m \in \mathbb{Z}_+$, $1 \leq i, j \leq m$, and $r, s \in \mathbb{Z}+\frac{1}{2}$.

We also use $\mathcal{C}l_{2m+1}$ to denote the Clifford algebra generated by

$$\psi_i^{\pm}(r),\ \psi_{2m+1}(r),\ r \in \mathbb{Z}+\frac{1}{2},\ 1 \leq i \leq m$$

with the non-trivial relations

$$\begin{aligned} \left[\psi_i^{\pm}(r), \psi_k^{\mp}(s)\right]_+ &= \delta_{ik}\delta_{r+s,0}, \\ \left[\psi_{2m+1}(r), \psi_{2m+1}(s)\right]_+ &= \delta_{r+s,0}, \end{aligned}$$

where $1 \leq i, k \leq m$, $r, s \in \mathbb{Z}+\frac{1}{2}$.

Let $\mathcal{F}_{2m}$ be the irreducible $\mathcal{C}l_{2m}$-module generated by the cyclic vector $\mathbf{1}$ such that

$$\psi_i^{\pm}(r)\mathbf{1} = 0,\ \text{for } r > 0,\ 1 \leq i \leq m$$

and $\mathcal{F}_{2m+1}$ the irreducible $\mathcal{C}l_{2m+1}$-module generated by the cyclic vector $\mathbf{1}$ such that

$$\psi_i^{\pm}(r)\mathbf{1} = \psi_{2m+1}(r)\mathbf{1} = 0,\ \text{for } r > 0,\ 1 \leq i \leq m.$$

It is well-known [1, 2] that both $\mathcal{F}_{2m}$ and $\mathcal{F}_{2m+1}$ are vertex operator super algebras.

The main idea of [3] is to identify the even part $(\mathcal{F}_m)^{even}$ with the affine VOA $L_{\widehat{\mathfrak{so}}_m}(1,0)$ (cf. [3, Theorems 3.1 and 3.2]); however, the identification stated in [3] is not correct when m is small. The correct statement is as follows.

Theorem 1.1. *For $m \in \mathbb{Z}_{\geq 4}$, we have*

$$(\mathcal{F}_m)^{even} \cong L_{\widehat{\mathfrak{so}}_m}(1,0).$$

Moreover,

$$(\mathcal{F}_3)^{even} \cong L_{\widehat{\mathfrak{sl}}_2}(2,0) \qquad \textit{and} \qquad (\mathcal{F}_2)^{even} \cong V_{\sqrt{2}A_1}.$$

Therefore, the analysis in [3] is still valid when $m, n \geq 4$ but the cases for $n = 2$ and $n = 3$ require special treatments. In [3], the case for $n = 2$ was also studied (see [3, Sections 4.1.1 and 4.2.1]. In the following, we will provide the argument for the case $n = 3$.

1.2. The case $n = 3$ and $m \in \mathbb{Z}_{\geq 4}$

By using Table 3 of [4] and the fact that $\mathcal{F}_3^{even}$ is isomorphic to $L_{\widehat{\mathfrak{sl}_2}}(2,0)$, we have

$$L_{\widehat{\mathfrak{so}}_{3m}}(1,0) \supset L_{\widehat{\mathfrak{so}}_m}(3,0) \otimes L_{\widehat{\mathfrak{sl}}_2}(2m,0)$$

as a full subVOA.

For the even cases, we relabel the generators of the Clifford algebra $\mathcal{C}l_{6m}$ so that it is generated by $\psi_{ij}^{\pm}(r)$, $1 \leq i \leq m, 1 \leq j \leq 3$, $r \in \mathbb{Z} + \frac{1}{2}$, with the non-trivial relations

$$[\psi_{ij}^{\pm}(r), \psi_{kl}^{\mp}(s)]_+ = \delta_{ik}\delta_{jl}\delta_{r+s,0}, \tag{1.1}$$

where $1 \leq i, k \leq m, 1 \leq j, l \leq 3$, $r, s \in \mathbb{Z} + \frac{1}{2}$. We also set

$$\begin{aligned} \psi_{kj}(-\frac{1}{2})\mathbf{1} &= \frac{1}{\sqrt{2}}(\psi_{kj}^{+}(-\frac{1}{2})\mathbf{1} + \psi_{kj}^{-}(-\frac{1}{2})\mathbf{1}), \\ \psi_{m+k,j}(-\frac{1}{2})\mathbf{1} &= \frac{-\sqrt{-1}}{\sqrt{2}}(\psi_{kj}^{+}(-\frac{1}{2})\mathbf{1} - \psi_{kj}^{-}(-\frac{1}{2})\mathbf{1}), \end{aligned}$$

for $1 \leq k \leq m$, $1 \leq j \leq 3$. In this case, $C_{L_{\widehat{\mathfrak{so}}_{6m}}(1,0)}(L_{\widehat{\mathfrak{so}}_{2m}}(3,0))$ contains a highest weight vector

$$u^{(12)} = \prod_{k=1}^{2m}(\psi_{k1}(-\frac{1}{2}) + \sqrt{-1}\psi_{k2}(-\frac{1}{2}))\mathbf{1}$$

and it generates the $L_{\widehat{\mathfrak{sl}_2}}(4k,0)$-module $L_{\widehat{\mathfrak{sl}_2}}(4k,4k)$. Hence, we have the following result.

Lemma 1.2. *For $m \in \mathbb{Z}_{\geq 2}$, we have*

$$C_{L_{\widehat{\mathfrak{so}}_{6m}}(1,0)}(L_{\widehat{\mathfrak{so}}_{2m}}(3,0)) = L_{\widehat{\mathfrak{sl}_2}}(4m,0) + L_{\widehat{\mathfrak{sl}_2}}(4m,4m).$$

By the definition of G, we also have the following result.

Theorem 1.3. *For $m \in \mathbb{Z}_{\geq 2}$,*

$$C_{L_{\widehat{\mathfrak{so}}_{2m}}(1,0)^{\otimes 3}}(L_{\widehat{\mathfrak{so}}_{2m}}(3,0)) = (L_{\widehat{\mathfrak{sl}_2}}(4m,0) + L_{\widehat{\mathfrak{sl}_2}}(4m,4m))^G.$$

Similarly, by using Table 3 of [4], we can deduce that for $m \geq 2$,

$$C_{L_{\widehat{\mathfrak{so}}_{3(2m+1)}}(1,0)}(L_{\widehat{\mathfrak{so}}_{2m+1}}(3,0)) = L_{\widehat{\mathfrak{sl}}_2}(4m+2,0).$$

Thus, we have the following result.

Theorem 1.4. *For $m \geq 2$, we have*

$$C_{L_{\widehat{\mathfrak{so}}_{2m+1}}(1,0)^{\otimes 3}}(L_{\widehat{\mathfrak{so}}_{2m+1}}(3,0)) = L_{\widehat{\mathfrak{sl}}_2}(4m+2,0)^G.$$

References

1. A. J. Feingold, I. B. Frenkel, Classical affine Lie algebras, *Adv. Math.*, **56** (1985), 117-172.

2. A. J. Feingold, I. B. Frenkel, J. Ries, Spinor construction of vertex operator algebras, Triality, and $E_8^{(1)}$, Amer. Math. Soc., Providence, 1991.

3. C. Jiang and C. H Lam, Level-rank duality for vertex operator algebras of type B and D, *Bull. Inst. Math. Acad. Sin.* (N.S.) **14** (2019), no. 1, 31-54.

4. V. G. Kac, Pierluigi Möseneder Frajria, Paolo Papi, and Feng Xu, Conformal embedding and simple current extensions, *Int. Math. Res. Not.* 2015, no. 14, 5229–5288; arXiv: 1210.6602v1.