On Landis’ conjecture and related questions

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Abstract

In the late 60’s, E.M. Landis conjectured that if \( \Delta u + Vu = 0 \) in \( \mathbb{R}^n \) with \( \|V\|_{L^\infty(\mathbb{R}^n)} \leq 1 \) and \( \|u\|_{L^\infty(\mathbb{R}^n)} \leq C_0 \) satisfying \( |u(x)| \leq C \exp(-C|x|^{1+}) \), then \( u \equiv 0 \). Landis’ conjecture was disproved by Meshkov who constructed such \( V \) and nontrivial \( u \) satisfying \( |u(x)| \leq C \exp(-C|x|^\frac{4}{3}) \). He also showed that if \( |u(x)| \leq C \exp(-C|x|^{\frac{4}{3}+}) \), then \( u \equiv 0 \). It should be noted that both \( V \) and \( u \) constructed by Meshkov are complex-valued functions. It remains an open question whether Landis’ conjecture is true for real-valued \( V \) and \( u \). Landis’ conjecture is closely related to the estimate of the maximal vanishing order of \( u \) in a bounded domain. In this talk, I would like to discuss my recent joint work with Kenig and Silvestre on Landis’ conjecture in two dimensions and related problems.