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Conformally invariant boundary value problems and nonlocal operators

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Abstract

Under a spectral condition on a compact Riemannian manifold with boundary, one can conformally deform the metric in the interior to one of zero scalar curvature while fixing the induced metric on the boundary. Indeed, this can be done by minimizing Escobar's boundary Yamabe functional, and the mean curvature of the boundary with respect to the scalar flat metric computes the fractional Q-curvature Q_1; this circle of ideas helps one prove a sharp Sobolev trace inequality. I will describe generalizations of this idea to higher-order and fully nonlinear invariants. First, I will describe how deforming the Q-curvature in the interior allows one both to recover higher-order fractional Q-curvatures from Dirichlet-to-Neumann operators and to prove higher-order sharp Sobolev trace inequalities. Second, I will describe how one can deform some fully nonlinear invariants, such as the σ_k-curvatures, to zero via a Dirichlet principle. Third, I will describe a partial progress towards an Obata-type theorem; this is expected to lead to proofs of some fully nonlinear Sobolev trace inequalities.