

# The Paneitz energy in dimension three revisited

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## Abstract

Paneitz operator and  $Q$  curvature in dimension three behaves very different from dimensions at least four. The two obvious differences are: the  $Q$  curvature equation involves negative power; the Paneitz operator has a negative eigenvalue on the standard three sphere. Sobolev inequality for Paneitz energy are not straightforward any more. Paul and Zhu derived a sharp Sobolev inequality on the standard three sphere long time ago. Here we provide another approach in the spirit of subcritical approximation in the classical Yamabe problem. This approach involves a new way of doing symmetrization for higher order variational problems. We will also discuss necessary and sufficient conditions for the validity of Sobolev inequality for metrics other than standard sphere.