

Representation of Markov kernels

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Abstract

Given some probability space $(\Omega, \mathcal{A}, \mu)$, and given an orthonormal base in $\mathcal{L}^2(\mu)$, say $(f_0 = 1, f_1, \dots, f_n, \dots)$, a general question is the description all Markov kernels K such that $K(f_n) = \lambda_n f_n$. The problem of determining the possible sequences (λ_n) is known as the Markov Sequence Problem. It is completely solved for example for the sequence of Hermite polynomials, for the Gegenbauer ones, and more generally for the sequences of Jacobi polynomials.

When the basis has the so-called hypergroup property (HGP in short), that I shall describe in this talk, then the problem is easily solved, and this is the case in the above mentioned results for the Jacobi polynomials. However, it remains in general quite hard to assert this hypergroup property.

In the talk, we shall present a powerful scheme, introduced by E. Carlen, J. Geronimo and M. Loss, which easily provides the HGP property. We shall show how it applies to the Jacobi polynomial case, and we shall investigate extensions of this result to the case of some polynomial orthonormal bases for the Dirichlet laws on the simplex.