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The Laplacian on some round Sierpiński carpets and Weyl's asymptotics for its eigenvalues

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Abstract

The purpose of this talk is to present the speaker's recent research in progress on the construction of a "canonical" Laplacian on round Sierpiński carpets invariant with respect to certain Kleinian groups (i.e., discrete groups of Möbius transformations on $\widehat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$) and on Weyl's asymptotics for its eigenvalues. Here a *round Sierpiński carpet* refers to a subset of $\widehat{\mathbb{C}}$ homeomorphic to the standard Sierpiński carpet, such that its complement in $\widehat{\mathbb{C}}$ consists of disjoint open disks in $\widehat{\mathbb{C}}$.

The construction of the Laplacian is based on the speaker's preceding study of the simplest case of the *Apollonian gasket*, the compact fractal subset of $\mathbb{C}$ obtained from an ideal triangle (a triangle formed by mutually tangent three circles) by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. On this fractal, Teplyaev (2004) had constructed a canonical Dirichlet form as one with respect to which the coordinate functions on the gasket are harmonic, and the author later proved its uniqueness and discovered an explicit expression of it in terms of the circle packing structure of the gasket.

The expression of the Dirichlet form obtained for the Apollonian gasket in fact makes sense on general circle packing fractals, including round Sierpiński carpets, and defines (a candidate of) a "canonical" Laplacian on such fractals. When the circle packing fractal is the limit set (i.e., the minimum invariant non-empty compact set) of a certain class of Kleinian groups, some explicit combinatorial structure of the fractal is known and makes it possible to prove Weyl's asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah [Invent. Math. **187** (2012), 1–35].

The overall structure of the proof of Weyl's asymptotic formula is the same as in the case of the Apollonian gasket and is based on a serious application of Kesten's renewal theorem [Ann. Probab. **2** (1974), 355–386] to a certain Markov chain on the *"space of all possible Euclidean shapes"* of the fractal. There is, however, a crucial difficulty in the case of a round Sierpiński carpet; since it is *infinitely ramified*, i.e., the cells in its cellular decomposition intersect on infinite sets, it is highly non-trivial to show that the principal order term of the eigenvalue asymptotics is not affected by the cellular decomposition, namely by assigning the Dirichlet boundary condition on the boundary of the cells. This is achieved by utilizing (1) an upper bound on the heat kernel obtained from a version of the Nash inequality, and (2) the geometric property, noted by Bonk in [Invent. Math. **186** (2011), 559–665], that the circles $\{C_k\}_{k=1}^{\infty}$ in the round carpet are *uniformly relatively separated*: there exists $\delta \in (0, \infty)$ such that

$$\mathrm{dist}(C_j, C_k) \geq \delta \min\{\mathrm{rad}(C_j), \mathrm{rad}(C_k)\} \qquad \text{for any } j, k \geq 1 \text{ with } j \neq k.$$