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Weighted partition of a compact metric space, its hyperbolicity and Ahlfors regular conformal dimension

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Abstract

Successive divisions of compact metric spaces appear in many different areas of mathematics such as the construction of self-similar sets, Markov partition associated with hyperbolic dynamical systems, dyadic cubes associated with a doubling metric space. The common feature in these is to divide a space into a finite number of subsets, then divide each subset into pieces and repeat this process again and again. In this paper we generalize such successive divisons and call them partitions. Given a partition, we consider the notion of a weight function assigning a "size" to each piece of the partition. Intuitively we believe that a partition and a weight function should provide a "geometry" and an "analysis" on the space of our interest. In this paper, we are going to pursue this idea in three parts. In the first part, the metrizability of a weight function is shown to be equivalent to the Gromov hyperbolicity of the graph associated with the weight function. In the second part, the notions like bi-Lipschitz equivalence, Ahlfors regularity, the volume doubling property and quasisymmetry will be shown to be equivalent to certain properties of weight functions. In particular, we find that quasisymmetry and the volume doubling property are the same notion in the world of weight functions. In the third part, a characterization of the Ahlfors regular conformal dimension of a compact metric space is given as the critical index p of p-energies associated with the partition and the weight function corresponding to the metric.