

# Generalized conservation property of Brownian motion with killing inside

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## Abstract

Khasminskii's theorem says that Brownian motion on a weighted Riemannian manifold  $M$  satisfies the conservation property if and only if  $M$  enjoys the following property:

( $K$ ) For some/any  $\alpha > 0$ , any bounded solutions for  $(\alpha - \Delta)u = 0$  are trivial.

In this talk, we study a similar characterization for the diffusion process  $(\mathbb{X}_t)_{t>0}$  on  $M$  generated by  $L = \Delta - V$ , where  $V$  is a nonnegative nontrivial continuous function on  $M$ , called Brownian motion with killing inside.  $(\mathbb{X}_t)_{t>0}$  is never conservative; however, there are some pair of  $M$  and  $L$  which enjoys the property:

( $KL$ ) For some/any  $\alpha > 0$ , any bounded solutions for  $(\alpha - L)u = 0$  are trivial.

The main result of the talk is to propose a “generalized conservation property” ( $GCP$ ) for  $(\mathbb{X}_t)_{t>0}$  such that  $(\mathbb{X}_t)_{t>0}$  satisfies ( $GCP$ ) if and only if ( $KL$ ) is true. We will also give a characterization of ( $GCP$ ) for  $(\mathbb{X}_t)_{t>0}$  in terms of the classical conservation property of Brownian motion of a different weight on  $M$ . The main results in this talk were obtained in a joint work with Marcel Schmidt at Jena University.