

Generalized conservation property of Brownian motion with killing inside

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Abstract

Khasminskii's theorem says that Brownian motion on a weighted Riemannian manifold M satisfies the conservation property if and only if M enjoys the following property:

(K) For some/any $\alpha > 0$, any bounded solutions for $(\alpha - \Delta)u = 0$ are trivial.

In this talk, we study a similar characterization for the diffusion process $(\mathbb{X}_t)_{t>0}$ on M generated by $L = \Delta - V$, where V is a nonnegative nontrivial continuous function on M , called Brownian motion with killing inside. $(\mathbb{X}_t)_{t>0}$ is never conservative; however, there are some pair of M and L which enjoys the property:

(KL) For some/any $\alpha > 0$, any bounded solutions for $(\alpha - L)u = 0$ are trivial.

The main result of the talk is to propose a “generalized conservation property” (GCP) for $(\mathbb{X}_t)_{t>0}$ such that $(\mathbb{X}_t)_{t>0}$ satisfies (GCP) if and only if (KL) is true. We will also give a characterization of (GCP) for $(\mathbb{X}_t)_{t>0}$ in terms of the classical conservation property of Brownian motion of a different weight on M . The main results in this talk were obtained in a joint work with Marcel Schmidt at Jena University.