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Exponential self-similar mixing by incompressible flows

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Abstract

I will address the problem of the optimal mixing of a passive scalar under the action of an incompressible flow in two space dimensions. The scalar solves the continuity equation with a divergence-free velocity field which satisfies a bound in the Sobolev space $W^{s,p}$, where $s \geq 0$ and $1 \leq p \leq \infty$. The mixing properties are given in terms of a characteristic length scale, called the mixing scale. We consider two notions of mixing scale, one functional, expressed in terms of the homogeneous Sobolev norm $\dot{H}^{-1}$, the other geometric, related to rearrangements of sets. We study rates of decay in time of both scales under self-similar mixing. For the case $s = 1$ and $1 \leq p \leq \infty$ (including the Lipschitz case, and the case of physical interest of enstrophy-constrained flows), we present examples of velocity fields and initial configurations for the scalar that saturate the exponential lower bound established in previous works for the decay in time of both scales. We also obtain several consequences for the geometry of regular Lagrangian flows associated to Sobolev velocity fields and for the loss of regularity for continuity equations with non-Lipschitz velocity field. The talk will be based on joint works with G. Alberti (University of Pisa, Italy) and A. L. Mazzucato (Penn State).