

# Non-uniqueness for the incompressible Euler equations up to Onsager's critical exponent

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## Abstract

Our work is related to Onsager's conjecture, proven by Isett and refined by Buckmaster, De Lellis, Szekelyhidi and Vicol, according to which below Hölder regularity  $1/3$  there exist solutions of the incompressible Euler equations which dissipate the total kinetic energy of the flow. In a series of papers we deal with the associated initial value problem, with regularity below Onsager's critical exponent. Although in a smooth setting dissipation of the total kinetic energy implies uniqueness of solutions with the same initial datum, we are able to show that there exists a dense set of Hölder  $1/3$ -initial data, each admitting infinitely many Hölder  $1/3$  dissipative solutions of the Euler equations. In particular, an instance of the  $h$ -principle holds for the incompressible Euler equations below the critical exponent, similarly to what happens for  $C^1$  isometric embeddings of Riemannian manifolds in  $\mathbb{R}^n$ . Moreover, in order to prove the density in  $L^2$  of such wild initial data we introduce a family of stationary solutions of the Euler equations, the so called Mikado flows, which proved to be the crucial ingredient in Isett's proof of the Onsager's conjecture. Some of the results are a joint work with E. Runa and L. Szekelyhidi.