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Approximation of optimal control problems for conservation laws via vanishing viscosity and relaxation

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Abstract

We consider an optimal control problem for a system of conservation laws in one space dimension, where the control acts through the initial datum, namely

$$\min_{\bar{u}\in\mathcal{D}} \int_{\mathbb{R}} J(u(T,x))\,dx\,, \tag{1}$$

where $J : \mathbb{R}^n \to \mathbb{R}$ is smooth and strictly convex, $T > 0$, $\mathcal{D} \subset L^1(\mathbb{R})$, and $u = u(t,x) \in \mathbb{R}^n$ is the solution to

$$\begin{cases} \partial_t u + \partial_x f(u) = 0 \\ u(0,x) = \bar{u}(x)\,. \end{cases} \tag{2}$$

We study problem (1)-(2) by analysing the optimal solutions to two kinds of approximate control problems,

$$\begin{cases} \displaystyle\min_{\bar{u}^\varepsilon\in\mathcal{D}^\varepsilon} \int_{\mathbb{R}} J\big(u^\varepsilon(T,x)\big)\,dx \\ \partial_t u^\varepsilon + \partial_x f(u^\varepsilon) = \varepsilon\partial_{xx}u^\varepsilon \\ u(0,x) = \bar{u}^\varepsilon(x)\,, \end{cases} \qquad \begin{cases} \displaystyle\min_{\bar{u}^\varepsilon\in\mathcal{D}^\varepsilon} \int_{\mathbb{R}} J\big(u^\varepsilon(T,x)\big)\,dx \\ \partial_t u^\varepsilon + \partial_x f(u^\varepsilon) = \varepsilon\big(\partial_{xx}u^\varepsilon - \partial_{tt}u^\varepsilon\big) \\ u(0,x) = \bar{u}^\varepsilon(x)\,, \end{cases} \tag{3}$$

where $\mathcal{D}^\varepsilon \subset \mathcal{D}$. We established necessary conditions for optimality for these problems, and study the Γ-convergence to (1)-(2) as $\varepsilon \to 0^+$. The final goal is to derive a constructive algorithm to determine optimal solutions to (1)-(2) as limit of optimal solutions to (3).