International Workshop on Hyperbolic and Kinetic Problems: Theory and Applications
Institute of Mathematics, Academia Sinica, Taipei, Taiwan
July 10 - 14, 2018

Untangling of trajectories for non-smooth vector fields and Bressan's Compactness Conjecture

Stefano Bianchini, Paolo Bonicatto

Universität Basel
E-mail:**paolo.bonicatto@unibas.ch**

Abstract

Given $d \geq 1$, $T > 0$ and a vector field $\mathbf{b}\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$, we study the problem of uniqueness of weak solutions to the associated transport equation $\partial_t u + \mathbf{b} \cdot \nabla u = 0$ where $u\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}$ is an unknown scalar function. In the classical setting, the method of characteristics is available and provides an explicit formula for the solution of the PDE, in terms of the flow of the vector field $\mathbf{b}$. However, when we drop regularity assumptions on the velocity field, uniqueness is in general lost. In the talk we will present an approach to the problem of uniqueness based on the concept of Lagrangian representation. This tool allows to represent a suitable class of vector fields as superposition of trajectories: we will then give local conditions to ensure that this representation induces a partition of the space-time made up of disjoint trajectories, along which the PDE can be disintegrated into a family of 1-dimensional equations. We will finally show that if $\mathbf{b}$ is locally of class BV in the space variable, the decomposition satisfies this local structural assumption: this yields in particular the renormalization property for nearly incompressible BV vector fields and thus gives a positive answer to the (weak) Bressan's Compactness Conjecture. This is a joint work with S. Bianchini.