Embedded Willmore tori in three-manifolds with small area constraint

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Abstract

While there are lots of contributions on Willmore surfaces in the three-dimensional Euclidean space, the literature on curved manifolds is still relatively limited. One of the main aspects of the Willmore problem is the loss of compactness under conformal transformations. We construct embedded Willmore tori in manifolds with a small area constraint by analysing how the Willmore energy under the action of the Mobius group is affected by the curvature of the ambient manifold. The loss of compactness is then taken care of using minimisation arguments or Morse theory.
Boundary expansions for geometric PDEs

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Abstract

In some geometric problems, we need to discuss the asymptotic expansions of solutions near boundary and estimate the remainders. The list of such problems includes the singular Yamabe problem, the regularity of minimal surface near the asymptotic infinity in the hyperbolic space and the complete Kahler-Einstein metrics in strictly pseudo-convex domains. Usually, the underlying equations become degenerate along boundary. In this talk, we present a PDE approach for remainder estimates, which are referred to as the polyhomogeneity, and for the global regularity up to boundary.
Fourth order Q-curvature equation in conformal geometry

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Abstract

In dimensions greater than two, the fourth order Q-curvature equation has some progress in the last couple of years. I plan to talk about the Green’s function of the fourth order GJMS operator in dimensions other than four, and its associated consequence: a strong maximum principle for this equation. (joint work with Fengbo Hang).
The geometry of pseudohermitian submanifolds of the Heisenberg groups

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Abstract

The Heisenberg geometry is a kind of Klein geometry. The Heisenberg group is the model (flat) space of pseudohermitian manifolds, with the group of pseudohermitian transformations as the symmetry group. A fundamental problem in Klein geometry is to ask when two submanifolds are congruent with each other. In this talk, we will focus on pseudohermitian submanifolds of the Heisenberg group. We will introduce some basic properties and obtain a complete set of invariants for pseudohermitian submanifolds, as well as the local embedding theorem if time is enough. Our method is the Cartan’s method of moving frame and calculus on Lie groups.
Hamiltonian diffeomorphisms and hyperKahler metrics

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Abstract

Hitchin recently proposed an analog of the Hitchin component for SL(∞, R). I will explain
a construction of this component. This involves the notion of folded hyperKahler metrics on
pseudoconcave cotangent domains.
Convexity and embeddedness for hypersurfaces in hyperbolic space

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Abstract

It was conjectured that, except for covering maps of equidistant surfaces in hyperbolic 3-space, every complete, noncompact, nonnegatively curved, immersed hypersurface in hyperbolic space is properly embedded. It said that Gromov had proposed a proof for this conjecture except for surfaces. In this talk we will use the horospherical metrics for nonnegatively curved hypersurfaces in hyperbolic space to prove this conjecture in all dimensions. This is a joint work with Vincent Bonini and Shiguang Ma.
Compactness of conformally compact Einstein manifolds in dimension 4

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Abstract

We show some compactness result of 4-dimensional conformally compact Einstein manifolds under the suitable assumptions on the conformal infinity and on some suitable conformal invariants. It is a joint work with Alice Chang.
Gradient estimate and spectral rigidity for nonlinear Laplacian

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Abstract

The talk primarily concerns the analysis of the p-Laplacian. We will first explain a sharp gradient estimate for the positive eigenfunctions. As an application, we then discuss the rigidity of manifolds with maximal principal eigenvalue. Finally, we will mention an Agmon type estimate.
Willmore Minmax Surfaces and the Cost of the Sphere Eversion

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Abstract

We develop a general Minmax procedure in Euclidian spaces for constructing Willmore surfaces of non zero indices. We implement this procedure to the Willmore Minmax Sphere Eversion in the 3 dimensional euclidian space in order to compute the cost of this famous eversion.
Sharp local decay estimates for the Ricci flow on surfaces

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Abstract

There are many tools available when studying 2D Ricci flow, equivalently the logarithmic fast diffusion equation, but one has always been missing: how do you get uniform smoothing estimates in terms of local $L^1$ data, i.e. in terms of local bounds on the area. The problem is that the direct analogue of the geometrically less-useful $L^p$ smoothing estimates for $p > 1$ are simply false. In this talk I will explain this problem in more detail, and show how to get around it with a new local decay estimate. I also plan to sketch the proof and/or give some applications. Joint work with Hao Yin.
Asymptotically Hyperbolic Initial Data for Einstein’s Equation

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Abstract

If we split spacetime into a product $M \times \mathbb{R}$, where $M$ is a 3-manifold, we can view Einstein’s equation for a gravitational field as a coupled nonlinear evolution equation for a time-dependent Riemannian metric and second fundamental form. Initial data for this equation consist of a Riemannian metric $g$ on $M$ together with a symmetric 2-tensor field $K$ representing the second fundamental form. Unlike the linear wave equation, these initial data cannot be specified independently, but must satisfy a necessary condition (based on the Gauss and Codazzi equations) called the Einstein constraint equations. A good approach to parametrizing the possible solutions to Einstein’s equations is to try to parametrize solutions to the constraint equations.

I will talk about an approach to parametrizing asymptotically hyperbolic solutions to the constraint equations, which model initial data along a spacelike surface asymptotic to the forward light cone. Such initial data are natural candidates for constructing spacetimes that have conformal compactifications at forward timelike infinity, which should be extremely useful for studying gravitational radiation and black holes. There is an important boundary condition at infinity, called the shear-free condition, that must be satisfied in order for the initial data to have a forward evolution that admits a conformal compactification. Many asymptotically hyperbolic solutions to the constraint equations have been constructed, but previously none of them have been able to ensure that the shear-free condition is satisfied.

I will describe joint work with Paul Allen, Jim Isenberg, and Iva Stavrov-Allen, in which we parametrize shear-free, asymptotically hyperbolic, constant-mean-curvature solutions to the constraint equations. Ensuring that the shear-free condition holds necessitates working in new spaces of metrics we call “weakly asymptotically hyperbolic.” As I will explain, we expect these spaces to be useful in many other natural problems on asymptotically hyperbolic manifolds.
Szegö kernel, Morse inequalities and Kodaira embedding theorem on CR manifolds with $S^1$ action

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Abstract

Let $X$ be a compact CR manifold with a transversal CR locally free $S^1$ action and let $L^k$ be the $k$-th tensor power of a rigid positive CR line bundle $L$ over $X$. Without any Levi curvature assumption, we prove that the partial Szegö kernel admits a full asymptotic expansion and by using these asymptotics, we establish Kodaira embedding theorem on CR manifolds with transversal CR locally free $S^1$ action. The results are joint with Xiaoshan Li and George Marinescu.
The soliton solutions for mean curvature flow

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Abstract

Mean Curvature Flow (MCF) is a canonical way to deform sub-manifolds to minimal sub-manifolds. It also improves the geometric properties of sub-manifolds along the flow. Unfortunately, singularities may occur and cause obstructions to continue the flow. Isolated singularity models on soliton solutions that include self-similar solutions and translating solutions. Together with Wang, and Joyce-Tsui, respectively, we constructed many new examples of soliton solutions with special character for Lagrangian mean curvature flow. These examples demonstrate some important properties and later are proved to be unique with the given property. In this talk, I will discuss these examples and some related results.
Finite total $Q$-curvature on a locally conformally flat manifold

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Abstract

In this talk, we will discuss locally conformally flat manifolds with finite total curvature. We prove that for such a manifold, the integral of the $Q$-curvature equals an integral multiple of a dimensional constant. This shows a new aspect of the $Q$-curvature on noncompact complete manifolds. It provides further evidence that $Q$-curvature controls geometry as the Gaussian curvature does in two dimension on locally conformally flat manifolds. This is joint work with Zhiqin Lu.
On Metrics with Non-Negative Ricci Curvature on Convex Three-Manifolds

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Abstract

We prove that the space of smooth Riemannian metrics on the three-ball with non-negative Ricci curvature and strictly convex boundary is path-connected. As an application, using results of Maximo, Nunes, and Smith, we show the existence of properly embedded free boundary minimal annulus on any three-ball with non-negative Ricci curvature and strictly convex boundary.
Integral Kahler Invariants and the Bergman kernel asymptotics for line bundles

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Abstract

On a compact Kahler manifold, one can define global invariants by integrating local invariants of the metric. Assume that a global invariant thus obtained depends only on the Kahler class. Then we show that the integrand can be decomposed into a Chern polynomial (the integrand of a Chern number) and divergences of one forms, which do not contribute to the integral. We apply this decomposition formula to describe the asymptotic expansion of the Bergman kernel for positive line bundles and to show that the CR Q-curvature on a Sasakian manifold is a divergence. This is a joint work with Spyros Alexakis (U Toronto).
Asymptotics of Cornered Poincaré-Einstein Metrics

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Abstract

A cornered asymptotically hyperbolic metric is a metric defined on a space with two boundary components meeting in a codimension two corner. Near one of the boundary components the metric is asymptotically hyperbolic and near the other it is smooth. I will discuss a normal form near the corner for such metrics, and their formal asymptotics in the case the metric is Einstein with negative scalar curvature and satisfies a CMC umbilic boundary condition on the finite boundary. This is work of my finishing Ph.D. student Stephen McKeown.
Uniqueness problems and results in CR geometry

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Abstract

We will first discuss a remarkable formula discovered by Jerison and Lee to classify constant scalar curvature pseudohermitian structures on the sphere. We show that the formula is valid in the wider context of Einstein pseudohermitian manifolds. As an application we prove a uniqueness result that generalizes the theorem of Jerison and Lee. Then we will talk about alternative approaches which may yield stronger uniqueness results. Partial results and open problem will be presented.
A Penrose Transform for Nonembeddable CR Structures on $S^3$

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Abstract

In this talk, we give an explicit correspondence between the space of nonembeddable CR structures on $S^3$ and the space of holomorphic projective structures on $\mathbb{B}^2 \subset \mathbb{C}^2$. The space of nonembeddable CR structures near the standard structure on $S^3$ can be classified by two conjugate holomorphic functions of two variables. These structures can be naturally extended to deformations of the standard complex structure on $\mathbb{P}^2 \setminus \mathbb{B}^2$ which agree with the standard complex structure on the $\mathbb{P}^1$ at $\infty$. Since the resulting pseudoconcave surface has rational curves with self intersection one, Hitchin’s nonlinear Penrose transform identifies the space of rational curves as a second complex manifold with a holomorphic projective structure defined by the incidence relation between rational curves. We explicitly exhibit this correspondence between pseudoconcave manifolds and projective structures as a deformation of the standard duality between lines and planes in $\mathbb{P}^2$; in doing so, we demonstrate how the conjugate holomorphic functions describing the deformation of the CR structure simultaneously define a deformation of the flat projective structure on $\mathbb{B}^2$. 