Workshop on Complex Hyperbolicity, Function Fields and Non-Archimedean Arithmetic

June 8-12, 2026

June 8, 9:30-10:30

Junjiro Noguchi (University of Tokyo)

Around Manin-Grauert in higher dimensions

In 1960 (Publ. IHES) S. Lang proposed an analogue of Mordell's Conjecture over function fields. It was solved affirmatively by Yu. Manin in 1963 (Izves.) and H. Grauert in 1965 (Publ. IHES) by different methods. The result says:

Theorem 1. *Let $X \to R$ be an algebraic fiber space of compact smooth curves of genus ≥ 2 over an affine algebraic smooth curve R. If there are infinitely many sections in X/R, then $X \cong R \times X_{t_0}$ with $t_0 \in R$ and there are only finitely many non-constant sections in $R \times X_{t_0}$.*

Around 1970 S. Kobayashi introduced the notion of (Kobayashi) hyperbolicity, and then Lang formulated a number of interesting problems on rational points in terms of Kobayashi hyperbolicity in higher-dimensional algebraic varieties as well as the relative case over function fields.

In this talk, I will begin by recalling the cases of general-dimensional algebraic varieties with ample cotangent bundle (N. 1981) and hyperbolic fiber spaces (N. 1985, 1992). The proofs consist of two steps:

(i) (Boundedness) The degrees (heights) of sections (rational points) are bounded, so that the moduli Σ of all sections is a compact complex space.

(ii) (Rigidity) Each section is rigid in the sense of deformation, or $\dim \Sigma = 0$.

For the boundedness (i), it is necessary in general to assume the following boundary condition:

(BC) Let $X \to R$ be a compact hyperbolic fiber space with compactification $\overline{X} \to \overline{R}$. Then the inclusion $X \hookrightarrow \overline{X}$ is a hyperbolic embedding along the boundary $\overline{X}|_{\overline{R}\setminus R}$.

If the inner space X/R carries sufficient information, we do not need the boundary condition (BC); the case of ample $T^*_{X/R}$ is such a case. If a topological rigidity for sections is known, we even have some geometric bounds for the number of sections without (BC) (not yet published).

From the viewpoint above, I would like to discuss some recent results by Xie–Yuan (2026) and Bartsch–Javanpeykar (2024) for ramified covers over abelian varieties relative over a curve. In this case (BC) is not necessary, and the Mordell–Weil (Lang–Néron) theorem plays an essential role.

For the constant case we know:

Theorem 2. [N.-Sunada (1982), de Franchis type] Let M, N be compact complex manifolds. If $\bigwedge^k T_M$ ($k \in \mathbf{N}$) is Grauert negative, there are only finitely many meromorphic mappings from N to M of which generic differential rank is $\geq k$.

From this viewpoint I discuss D. Brotbek's hyperbolic hypersurfaces in $\mathbf{P}^n$ (2017) with, roughly speaking, an ample jet differential bundle, which solved the Kobayashi Conjecture. For instance,

Theorem 3. *Let $X \subset \mathbf{P}^n$ be a Brotbek hypersurface, and let R be a compact smooth curve (in fact, any compact complex reduced space). Then there are only finitely many non-constant holomorphic maps from R into X.*

This does not hold under only the Kobayashi hyperbolicity assumption on X. I will also discuss a result by C. Mourougane (2012) for families of projective hypersurfaces of sufficiently high degree which move enough.

June 8, 11:00-12:30

Song-Yan Xie (Chinese Academy of Sciences)

A Second Main Theorem for Entire Curves Intersecting Three Conics

Abstract: We establish a Second Main Theorem for entire holomorphic curves $f : \mathbb{C} \to \mathbb{P}^2$ intersecting a generic configuration of three conics $C = C_1 + C_2 + C_3$ in the complex projective plane $\mathbb{P}^2$. Using invariant logarithmic 2-jet differentials with negative twists, we prove the estimate

$$T_f(r) \leqslant 5 \sum_{i=1}^{3} N_f^{[1]}(r, C_i) + o\big(T_f(r)\big) \qquad \|,$$

where $T_f(r)$ is the Nevanlinna characteristic function, and $N_f^{[1]}(r, C_i)$ is the 1-truncated counting function.

The key innovation of our approach is establishing new vanishing lemmas of the form

$$H^0\big(\mathbb{P}^2,\ E_{2,m}T^*_{\mathbb{P}^2}(\log C)\otimes \mathcal{O}_{\mathbb{P}^2}(-t)\big) = 0$$

for specific pairs (m, t), achieved by combining algebro-geometric arguments with computer-assisted computations through a mod-p reduction technique. This yields a systematic method for proving vanishing results for negatively twisted jet differentials—a key component in complex hyperbolic geometry.

Discussion Session: 30-minutes in the afternoon of the talk.

June 9, 9:30-10:30

Aaron Levin (Michigan State University)

Greatest common divisors and an unlikely intersection type conjecture

We discuss recent results on greatest common divisors in Diophantine approximation and a new geometric conjecture inspired by a careful study of these results. This is joint work with Keping Huang.

June 9, 11:00-12:00

Zheng Xiao (University of Colorado, Boulder)

Arithmetic Discriminant and GCD for Genus 2 Curves

Vojta's conjecture occupies a central position in the field of Diophantine approximation, providing profound insights into the distribution of rational and integral points. While the conjecture exists in both rational and algebraic forms, its general formulation remains inaccessible. Consequently, contemporary research often focuses on a refined version that substitutes the field discriminant with the arithmetic discriminant. Remarkably, establishing estimates for the arithmetic discriminant provides a direct pathway to proving the GCD conjecture on abelian surfaces. To demonstrate this relationship, we will examine the Jacobians of genus 2 curves as a primary illustrative case.

June 10, 9:30-10:30

Erwan Rousseau (Université de Brest)

Generalized irregularities and specialness

In joint works with S. Kebekus and F. Touzet, we consider irregularities defined by arbitrary (ramified) covers. I will describe their main properties, relations with specialness (in the sense of Campana) and new applications to hyperbolicity questions.

References: Irregularities of special C-pairs arXiv:2601.07318

June 10, 11:00-12:00

Amos Turchet (Roma Tre University)

Algebraic exceptional sets on surfaces of log general type

we will review the concept of exceptional sets appearing in conjectures of Lang and Vojta. We will then focus on recent joint work with Lucia Caporaso that completely describes the algebraic exceptional set for complements of three components divisors in surfaces.

June 11, 9:30-10:30

Laura Capuano (Roma Tre University)

Singular intersections of curves and subgroups in tori

It can be proved that, if X is an irreducible curve in a torus defined over a number field, then the set of all the points of X lying in a proper algebraic group is always infinite, even under the hypothesis that X is not contained in a proper algebraic group. In a joint work with F. Ballini and N. Ottolini we prove that, if one looks at the multiplicities of intersections, the set of points where this intersection is singular is a finite set. This statement, which fits in the general framework of problems of Unlikely Intersections, generalizes a previous result of Marché and Maurin for curves in a torus of dimension 2.

June 11, 11:00-12:20

Jackson Morrow (University of North Texas)

Arithmetic differential equations and unlikely intersections

Unlikely intersection problems occupy a central place in modern arithmetic geometry. One of the foundational results in this area is Raynaud's theorem resolving the Manin-Mumford conjecture, which asserts that if a closed subvariety of an abelian variety contains a Zariski-dense set of torsion points, then it is a translate of an abelian subvariety. In particular, if the subvariety contains no translates of abelian subvarieties, then its intersection with the torsion subgroup is finite, and one may ask for explicit bounds on its size. In this talk, I will describe a p-adic technique for producing such explicit bounds, founded in Buium's theory of arithmetic jet spaces and arithmetic differential equations. I will introduce this theory from first principles, beginning with the notion of a p-derivation and culminating in the definition of an arithmetic jet space. Along the way, I will discuss important results in the area, including Buium's theorem of the kernel–a p-adic analogue of Manin's theorem of the kernel. I will then describe joint work with Lance Edward Miller establishing an explicit bound on the intersection of a variety with ample cotangent bundle with the torsion subgroup of an abelian variety, generalizing Buium's quantitative Manin-Mumford theorem for hyperbolic curves. I will conclude by discussing future research directions, with an eye toward connections between this theory and techniques from complex hyperbolicity.

Discussion Session: 30-minutes in the afternoon of the talk.

References: See Page 4.

June 12, 9:30-10:30

Carlo Gasbarri (University of Strasbourg)

Rational and transcendental points on entire curves

Let X be a variety over defined over a number field K and let Z be a Zariski dense entire curve inside it. We will describe how to count the number of rational points inside Z in terms of the size of the height and the Nevanlinna Counting Function. We will explain how, the presence of some special kind of "generic" transcendence points in Z has an influence on the this number.

References:

(i) E. Bombieri, J. Pila. *The number of rational points on arcs and ovals*, Duke Math. J. 59 (1989), 337–357.
(ii) C. Gasbarri, *Rational points of bounded height on entire curves*, arXiv:2504.06665
(iii) C. Gasbarri, *Rational versus transcendental points on analytic Riemann surfaces*, Manuscripta Math.169 (2022), no.1-2, 77–105.

June 12, 11:00-12:00

Damian Brotbek (University of Lorraine)

Logarithmic degree of irrationality

A classical invariant of a projective curve C is its gonality, namely the lowest possible degree of a dominant map $f : \mathbb{C} \to \mathbb{P}^1$. In higher dimension, one can consider the degree of irrationality of any projective variety X of dimension n, this is the lowest possible degree of a dominant rational map $f : X \dashrightarrow \mathbb{P}^n$. This degree of irrationality has recently been at the center of a lot of investigations.

In this talk I will explain how to define a logarithmic analogue of the degree of irrationality, and give some first results and example concerning this invariant. This is a joint work in progress with Gianluca Pacienza.

REFERENCES FOR “ARITHMETIC DIFFERENTIAL EQUATIONS AND UNLIKELY INTERSECTIONS”

JACKSON S. MORROW

In this short document, we compile references for the author's talk “Arithmetic differential equations and unlikely intersections” at the International Workshop “Complex Hyperbolicity, Function fields and Non-archimedean Arithmetic” (in Taiwan).

Basics of Buium's theory of p-derivatives. Below are references, which provide down-to-earth and easily accessible references for Buium's theory of p-derivatives.

- [Bui97]: While not one of the first works on p-derivatives, Buium shows that p-derivatives are natural in a sense; this work only requires basic knowledge of algebraic number theory.
- [Bui15]: This is one of Buium's research statements. It provides a large amount of motivation for the area and in Section 2, highlights many important results in the area.

Applications of p-derivatives to arithmetic geometry. There are many fantastic applications of p-derivatives which the author will highlight in his talk.

- [Bui95]: In this work, Buium proves an arithmetic analogue of Manin's theorem of the kernel. This article represents the starting point for the theory of p-derivatives.
- [Bui96]: In this work, Buium proves a quantitative Manin–Mumford for curves.
- [MM25]: In this work, the author with Lance Edward Miller prove a quantitative prime-to-p Manin–Mumford for varieties with ample cotangent bundle.
- [BP09]: In this work, Buium–Poonen prove an unlikely intersection result concerning the intersection of the canonical lift locus and a finite rank subgroup in the setting of a modular correspondence using Buium's theory of δ-modular forms. This is an instance of a so called reciprocity theorem in the δ-setting,which has also been studied in [BV22].

There are other applications of this theory, which the author will probably not be able to cover in the talk but are worth mentioning.

- [BS19]: In this work, Borger–Saha relate Buium's theory of p-derivatives to the crystalline cohomology of abelian schemes and also deduce properties of the group of δ-characters on an abelian scheme.
- [BM23a]: In this work, Buium–Miller use the theory of δ-functions to attach a perfectoid space to a smooth scheme over a discrete valuation ring with an algebraically closed residue field of characteristic $p > 0$.

Further reading on p-derivatives. A sufficiently motivated learner may want to gain a deeper understanding of δ-geometry and arithmetic differential equations. We recommend the following foundational texts.

- [Bui05]: This is the textbook on the theory of arithmetic differential equations and δ-geometry from Buium.

- [BPS23]: This work describes a more functorial construction of Buium's arithmetic jet spaces, which resembles the classical approach of jet spaces, where one replaces maps from $\operatorname{Spec} \mathbb{C}[\varepsilon]/(\varepsilon^n)$ with maps from the formal spectrum of the n-th truncated Witt vectors.
- [BM23b]: This work pushes the theory of δ-geometry to the ramified setting. Many of the ideas remain the same however there are now multiple p-derivatives in play; the reader should view this work as analogous to work in a partial differential field verse an ordinary differential field.

References

[BM23a] Alexandru Buium and Lance Edward Miller, *Perfectoid spaces arising from arithmetic jet spaces*, Amer. J. Math. **145** (2023), no. 1, 287–334. MR 4545848

[BM23b] ______, *Purely arithmetic PDEs over a p-adic field: δ-characters and δ-modular forms*, Memoirs of the European Mathematical Society, vol. 6, EMS Press, Berlin, 2023. MR 4631294

[BP09] Alexandru Buium and Bjorn Poonen, *Independence of points on elliptic curves arising from special points on modular and Shimura curves. II. Local results*, Compos. Math. **145** (2009), no. 3, 566–602. MR 2507742

[BPS23] Alessandra Bertapelle, Emma Previato, and Arnab Saha, *Arithmetic jet spaces*, J. Algebra **623** (2023), 127–153. MR 4556144

[BS19] James Borger and Arnab Saha, *Isocrystals associated to arithmetic jet spaces of abelian schemes*, Adv. Math. **351** (2019), 388–428. MR 3952574

[Bui95] Alexandru Buium, *Differential characters of abelian varieties over p-adic fields*, Invent. Math. **122** (1995), no. 2, 309–340. MR 1358979

[Bui96] ______, *Geometry of p-jets*, Duke Math. J. **82** (1996), no. 2, 349–367. MR 1387233

[Bui97] ______, *Arithmetic analogues of derivations*, J. Algebra **198** (1997), no. 1, 290–299. MR 1482984

[Bui05] ______, *Arithmetic differential equations*, Mathematical Surveys and Monographs, vol. 118, American Mathematical Society, Providence, RI, 2005. MR 2166202

[Bui15] ______, *Differential calculus with integers*, Arithmetic and geometry, London Math. Soc. Lecture Note Ser., vol. 420, Cambridge Univ. Press, Cambridge, 2015, pp. 139–187. MR 3467122

[BV22] Alexandru Buium and Adrian Vasiu, *The δ-invariant theory of Hecke correspondences on $\mathcal{A}_g$*, Preprint, arXiv:2208.00286 (July 30, 2022).

[MM25] Lance Edward Miller and Jackson S. Morrow, *Higher dimensional geometry of p-jets*, Preprint, arXiv:2510.00336 (September 30, 2025).

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