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0-TIGHT COMPLETELY 0-SIMPLE SEMIGROUPS

BY

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Abstract

A semigroup is 0-*tight* if each of its congruences is uniquely determined by each of the congruence classes which do not contain zero. We classify finite 0-tight rectangular 0-bands, and characterize 0-tight completely 0-simple semigroups. Finally, we obtain correspoding results about tight completely simple semigroups.

1. Introduction

Throughout this paper we shall use the terminology and notation of Howie [2]. We recall several definitions: a semigroup S is called *0-simple* if, for any $a, b \in S \setminus \{0\}$, there exist $x, y \in S$ such that $xay = b$. A *completely* 0-*simple semigroup* S is a 0-simple semigroup such that every idempotent z of S has the property that $zf = fz = f \neq 0$ implies $z = f$. The following result is due to Rees [6]. Every completely 0-simple semigroup is isomorphic to a certain Rees matrix semigroup. This construction has led to an extensive study of congruences on completely 0-simple semigroups (see [3], [5]).

We begin with an analysis of congruences on a particular finite completely 0-simple semigroup. The result turns out to be the structure theorem for finite 0-tight rectangular 0-bands (see Theorem 2.3 in [8]). We recall that a rectangular 0-band is a semigroup $(I \times \Lambda) \cup \{0\}$ whose product is given in

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The results of this paper are due to [8].

terms of a $\Lambda \times I$ matrix $P = (p_{\lambda i})$ with entries in $\{0, 1\}$ as follows:

$$(i, \lambda)(j, \mu) = \begin{cases} (i, \mu) & \text{if } p_{\lambda j} = 1 \\ 0 & \text{if } p_{\lambda j} = 0 \end{cases}$$

$$(i, \lambda)0 = 0(i, \lambda) = 00 = 0,$$

where P is regular, in the sense that no row or column of P consists entirely of zeros. The term *tight* was introduced by Schein [7]. A semigroup is called 0-*tight* if each of its congruences is uniquely determined by each of the congruence classes which do not contain zero. Using this structure theorem we then describe a characterization of 0-tight rectangular 0-bands.

The main aim of Section 3 is to characterize 0-tight completely 0-simple semigroups. A completely 0-simple semigroup $\mathcal{M}^0[G; I, \Lambda; P]$ is 0-tight if and only if the rectangular 0-band $(I \times \Lambda) \cup \{0\}$ is 0-tight. For the notation $\mathcal{M}^0[G; I, \Lambda; P]$, see p. 88 in [1].

By a similar approach to the study of 0-tight completely 0-simple semigroups, we investigate further tight completely simple semigroups. We recall that a semigroup S is called *completely simple* if S has no proper ideals and every idempotent z of S has the property that $zf = fz = f$ implies $z = f$. A semigroup is called *tight* if each of its congruences is uniquely determined by each of the congruence classes (see [7]). Our last result of Section 3 is a characterization of tight completely simple semigroups.

2. 0-Tight Rectangular 0-Bands

Every semigroup S with zero has exactly one of the following properties:

(1) S is 0-tight.

(2) No congruence on S except $S \times S$ is uniquely determined by each of its congruence classes which do not contain zero.

(3) There exists a congruence $\rho \neq S \times S$ on S such that ρ is uniquely determined by each of its congruence classes which do not contain zero. Also, there exist two congruences on S which have the same congruence class that does not contain zero.

A semigroup with the property in (2) is called 0-*tight-free*. A semigroup with the property in (3) is of the *third type*.

Now we proceed to the classification of finite rectangular 0-bands. Suppose S is a rectangular 0-band with $I = \{1, \ldots, m\}$ and $\Lambda = \{1, \ldots, n\}$. In [2] an equivalence relation $\mathcal{E}_I$ on I is defined by the rule that

$$(i, j) \in \mathcal{E}_I \text{ if } \{\lambda \in \Lambda : p_{\lambda i} = 0\} = \{\lambda \in \Lambda : p_{\lambda j} = 0\}.$$

The relation $\mathcal{E}_I$ is related to a unique partition of m in the following way: there are r $\mathcal{E}_I$-equivalence classes and m_i-element $\mathcal{E}_I$-classes for $0 \le i \le r \le |I|$ if

$$m = m_1 + \cdots + m_r, \tag{1}$$

where $m_1, \ldots, m_r \in \mathbb{N}$ and $m_1 \ge \cdots \ge m_r$.

Similarly, an equivalence relationan $\mathcal{E}_\Lambda$ on Λ is given by the rule that

$$(\lambda, \mu) \in \mathcal{E}_\Lambda \text{ if } \{i \in I : p_{\lambda i} = 0\} = \{i \in I : p_{\mu i} = 0\}.$$

Also, the relation $\mathcal{E}_\Lambda$ is related to a unique partition of n in the following: there are s $\mathcal{E}_\Lambda$-equivalence classes and n_j-element $\mathcal{E}_\Lambda$-classes for $0 \le j \le s \le |\Lambda|$ if

$$n = n_1 + \cdots + n_s, \tag{2}$$

where $n_1, \ldots, n_s \in \mathbb{N}$ and $n_1 \ge \cdots \ge n_s$.

We recall that a proper congruence ρ on a completely 0-simple semigroup is defined by $0\,\rho = \{0\}$. Supppose ρ is a relation on $S \setminus \{0\}$. According to Lemma 3.5.6 in [2], every proper congruence $\rho \cup \{(0, 0)\}$ on S is defined by the rule that

$$(i, \lambda)\ \rho\ (j, \mu) \Leftrightarrow (i, j) \in \mathcal{S} \text{ and } (\lambda, \mu) \in \mathcal{T}, \tag{3}$$

where $\mathcal{S}$ and $\mathcal{T}$ are equivalences such that $\mathcal{S} \subseteq \mathcal{E}_I$ and $\mathcal{T} \subseteq \mathcal{E}_\Lambda$.

Note that partitions of m and n do not always give the equivalences $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$, respectively. In fact, $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$ are determined by the existence of a regular matrix P (see Example 2.1). Now we investigate the cases where proper congruences are not uniquely determined by each of their congruence classes which do not contain zero.

Case 1. Let $P = (p_{\lambda i})$. Suppose the equivalence relations $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$ correspond to partitions of m and n, respectively, as follows:

$$m = m_1 + \cdots + m_r \text{ and } n = n_1 + \cdots + n_s,$$

where $r, s \geq 2$.

We look at the case where $|I| > 2$ or $|\Lambda| > 2$. For $|I| > 2$, suppose $m_1, \ldots, m_r$ are not all equal to 1. Clearly, for every equivalence $\mathcal{S}_1 \neq I \times I$, there exists an equivalence $\mathcal{S}_2 \neq \mathcal{S}_1$ such that $i\mathcal{S}_1 = i\mathcal{S}_2$ for some $i \in I$ according to the selection rules for the counting problems.

Similarly, for $|\Lambda| > 2$, suppose $n_1, \ldots, n_s$ are not all equal to 1. For every equivalence $\mathcal{T}_1 \neq \Lambda \times \Lambda$, there exists an equivalence $\mathcal{T}_2 \neq \mathcal{T}_1$, and $\lambda\mathcal{T}_1 = \lambda\mathcal{T}_2$ for some $\lambda \in \Lambda$. Let ρ_t correspond to $(\mathcal{S}_t, \mathcal{T}_t)$ for $1 \leq t \leq 2$. We have

$$(i, \lambda)\rho_1 = (i, \lambda)\rho_2,$$

for some $(i, \lambda) \in I \times \Lambda$.

When $i\mathcal{S}_1 = \{i\}$ and $\lambda\mathcal{T}_1 = \{\lambda\}$, a one-element $\mathcal{S}_1$-class and a one-element $\mathcal{T}_1$-class give the existence of a one-element ρ_1-class (see Example 2.2).

Case 2. Suppose $r = 1 = s$ in case 1. Then there exists a proper congruence ρ which corresponds to $(I \times I, 1_\Lambda)$ and ρ is uniquely determined by each of its congruence classes which do not contain zero. However, there exists a congruence which has a one-element class that is not equal to $\{0\}$. Hence such a finite rectangular 0-bands is of the third type.

Remark. Suppose $n \geq |\Lambda| \geq 2$ and $m \geq |I| \geq 2$. When $r = 1$ and $s \geq 2$, or $r \geq 2$ and $s = 1$, we cannot find a regular matrix P. In other words, in each of these cases we cannot find a rectangular 0-band whose multiplication is in terms some regular matrix P. Now suppose $n \geq |\Lambda| \geq 2$ and $m \geq |I| \geq 2^{|\Lambda|}$. When $m_1 = \cdots = m_r = 1$ and $n_1 = \cdots = n_s = 1$, again we cannot find a regular matrix P.

So far we have discussed the main part of the following classification.

(1) If $|\Lambda| = 1$ and $|I| = 1$ or 2, then S is 0-tight.

(2) If $|\Lambda| = 1$ and $m \geq |I| \geq 3$, then S is of the third type.
(3) If $|\Lambda| = 2$ and $|I| = 2$, then S is 0-tight.
(4) If $|\Lambda| = 2$ and $|I| = 3$, then S can be 0-tight or 0-tight-free or of the third type.
(5) If $|\Lambda| = 2$ and $m \geq |I| \geq 4$, then S is either 0-tight-free or of the third type.
(6) If $|\Lambda| = n \geq 3$ and $|I| = n, \ldots, 2^n - 1$, then S can be 0-tight or 0-tight-free or of the third type.
(7) If $|\Lambda| = n \geq 3$ and $m \geq |I| \geq 2^n$, then S is either 0-tight-free or of the third type.

Note that there is a duality between $|\Lambda|$ and $|I|$. For example, in order to know in which category a rectangular 0-band with $|\Lambda| = 4$ and $|I| = 3$ is, we simply check the rectangular 0-band with $|\Lambda| = 3$ and $|I| = 4$. Here we point out the number $2^n - 1$ coincides with that in Corollary 2.2 in [4]. Next, we recall the Green's equivalence $\mathcal{H}$.

$$(a, b) \in \mathcal{H} \Leftrightarrow xa = b,\ yb = a,\ au = b,\ bv = a \ \text{ for some } x, y, u, v \in S^1. \quad (4)$$

Also, a semigroup S is called *congruence-free* if S has no congruences other than 1_S and $S \times S$. [2] is a reference for the classification of finite congruence-free semigroups. The following theorem is the structure theorem for finite 0-tight rectangular 0-bands.

Theorem 2.1. *Suppose $I = \{1, \ldots, m\}$ and $\Lambda = \{1, \ldots, n\}$ are finite sets. If $|I| \leq 2$ and $|\Lambda| \leq 2$, then a rectangular 0-band is 0-tight. In other cases a rectangular 0-band is 0-tight if and only if it is congruence-free with zero. Conversely, every finite 0-tight semigroup S with $\mathcal{H} = 1_S$ is isomorphic to one of this kind.*

Proof. First every congruence-free semigroup with zero is 0-tight. We show other cases apart from $|I| \leq 2$ and $|\Lambda| \leq 2$ by verifying the following equivalent statements on a finite rectangular 0-band S:

(1) The finite rectangular 0-band S is 0-tight.
(2) $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = 1_\Lambda$.
(3) No two columns and no two rows of the matrix P are identical.

We shall show that (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3).

(1) $\Rightarrow$ (2). Suppose $\mathcal{E}_I \neq 1_I$ or $\mathcal{E}_\Lambda \neq 1_\Lambda$. From (1) and (2) on page 3, it follows that $m_1, \ldots, m_r$ are not all equal to 1 or $n_1, \ldots, n_s$ are not all equal to 1. By case 1 on page 4, there exists a proper congruence which is not uniquely determined by each of its congruence classes that do not contain zero. It is a contradiction.

(2) $\Rightarrow$ (1). If $\mathcal{E}_I = 1_I$, then $\mathcal{S} = 1_I$. Also, $\mathcal{E}_\Lambda = 1_\Lambda$ implies that $\mathcal{T} = 1_\Lambda$. By (3) on page 3, S has only one proper congruence, and hence S is 0-tight.

(2) $\Rightarrow$ (3). Suppose column i and column j of the regular matrix P are identical. Then

$$\{\lambda \in \Lambda : p_{\lambda i} = 0\} = \{\lambda \in \Lambda : p_{\lambda j} = 0\}.$$

This gives $(i, j) \in \mathcal{E}_I$. It is a contradiction. Similarly, if $\mathcal{E}_\Lambda = 1_\Lambda$, then no two rows of the matrix P are identical.

(3) $\Rightarrow$ (2). Suppose no two columns of the matrix P are identical. Since P has entries in $\{0, 1\}$, we have $\mathcal{E}_I = 1_I$. Similarly, Suppose no two rows of the matrix P are identical. Since P has entries in $\{0, 1\}$, we have $\mathcal{E}_\Lambda = 1_\Lambda$. We are done. Conversely, first every 0-tight semigroup is 0-simple. Also, it is known that every finite 0-simple semigroup S with $\mathcal{H} = 1_S$ is isomorphic to a finite rectangular 0-band. The remaining part of the proof follows the classification of finite 0-tight rectangular 0-bands. □

Now we apply Theorem 2.1 to arbitrary rectangular 0-bands.

Theorem 2.2. *Suppose S is a rectangular 0-band. If $|I| \leq 2$ and $|\Lambda| \leq 2$, then S is 0-tight. If $|I| \geq 3$ or $|\Lambda| \geq 3$, then the following statements are equivalent:*

(1) *S is 0-tight.*
(2) *1_S is uniquely determined by each of its congruence classes which do not contain zero.*
(3) *$\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = 1_\Lambda$.*
(4) *S is congruence-free with zero.*

Proof. It suffices to show that (2) implies (3). Let us assume that $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda \neq 1_\Lambda$. Let i and λ be fixed elements in I and Λ, respectively. We consider $\mathcal{T}$-classes $\{\lambda\}$ and $\Lambda \setminus \{\lambda\}$. Suppose the congruence ρ corresponds to $(1_I, \mathcal{T})$. Then ρ and 1_S have the same congruence class $\{(i,\lambda)\}$. We are done. □

Example 2.2. Let $|I| = 2 = |\Lambda|$. We find out the reason why partitions of m and n do not always give the equivalences $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$, respectively.

Since P is regular, there are only 7 possible P's. Let P_1 be a 2×2 matrix which consists of all 1's. Suppose each of P_2 to P_5 is a 2×2 matrix which consists of one 0 and three 1's. Suppose each of P_6 to P_7 is a 2×2 matrix which consists of two 0's and two 1's with either 0's or 1's on the main diagonal.

We check the rectangular 0-band whose multiplication is in terms of P_1. Now $\mathcal{E}_I = I \times I = \{1,2\} \times \{1,2\}$ and $\mathcal{E}_\Lambda = \Lambda \times \Lambda = \{1,2\} \times \{1,2\}$. In this case, $m = 2 = n$. Let $\mathcal{S}_1 = 1_I$, and $\mathcal{S}_2 = I \times I$, and $\mathcal{T}_1 = 1_\Lambda$, and $\mathcal{T}_2 = \Lambda \times \Lambda$. Suppose ρ_{kt} corresponds to $(\mathcal{S}_k, \mathcal{T}_t)$, where $1 \leq k \leq 2$ and $1 \leq t \leq 2$. Then we obtain the following proper congruences which are uniquely determined by each of their congruence classes that do not contain zero.

(1) $\rho_{11} \cup \{(0,0)\}$, the identity congruence.
(2) $\rho_{12} \cup \{(0,0)\}$ has classes $\{0\}, \{(1,1),(1,2)\}, \{(2,1),(2,2)\}$.
(3) $\rho_{21} \cup \{(0,0)\}$ has classes $\{0\}, \{(1,1),(2,1)\}, \{(1,2),(2,2)\}$.
(4) $\rho_{22} \cup \{(0,0)\}$ has classes $\{0\}, \{(1,1),(1,2),(2,1),(2,2)\}$,

Next, we look at the rectangular 0-band whose multiplication is in terms of some P_f, where $2 \leq f \leq 7$. Then $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = 1_\Lambda$. Now $m = 1+1 = n$. For each $2 \leq f \leq 7$, we obtain one proper congruence which is uniquely determined by each of its congruence classes that do not contain zero. When $m = 1+1$ and $n = 2$, we cannot say $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = \Lambda \times \Lambda$. In other words, if $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = \Lambda \times \Lambda$, then we cannot find a regular matrix equal to some P_f for $1 \leq f \leq 7$.

Example 2.2. Let $m = 2+2 = n$. We give an example of a 0-tight-free rectangular 0-band. Now there are 2 two-element $\mathcal{E}_I$-classes and 2 two-element $\mathcal{E}_\Lambda$-classes. Here we only discuss one possible arrangement of elements of each class selected from the sets I and Λ.

Suppose $\mathcal{S}_1$-classes are $\{1,2\}, \{3,4\}$, and $\mathcal{S}_2$-classes are $\{1,2\}, \{3\}, \{4\}$, and $\mathcal{S}_3$-classes are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$. Suppose $\mathcal{T}_1$-classes are $\{1,3\}, \{2,4\}$, and $\mathcal{T}_2$-classes are $\{1,3\}$, $\{2\}$, $\{4\}$, and $\mathcal{T}_3$-classes are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$.

Also, let ρ_{kt} correspond to $(\mathcal{S}_k, \mathcal{T}_t)$, where $1 \leq k \leq 3$ and $1 \leq t \leq 3$. We obtain the following result:

(1) $(1,3)\,\rho_{11} = (1,3)\,\rho_{12} = (1,3)\,\rho_{21} = (1,3)\,\rho_{22} = \{(1,1),(1,3),(2,1),(2,3)\}$.
(2) $(1,1)\,\rho_{13} = (1,1)\,\rho_{23} = \{(1,1),(2,1)\}$.
(3) $(1,1)\,\rho_{31} = (1,1)\,\rho_{32} = \{(1,1),(1,3)\}$.
(4) $(4,4)\,\rho_{23} = (4,4)\,\rho_{33} = \{(4,4)\}$.

3. 0-Tight Completely 0-Simple Semigroups

The Green's equivalence $\mathcal{H}$ (see (4) on page 5) plays a major role in analyzing 0-tight completely 0-simple semigroups. Notice that $\mathcal{H}$ is a congruence on a completely 0-simple semigroup. We consider a morphism which maps a rectangular 0-band Y into the quotient semigroup $S/\mathcal{H}$ by sending (i, λ) into $\mathcal{H}$-class $H_{i\lambda}$, where $H_{i\lambda}$ is either a group or ${H_{i\lambda}}^2 = 0$

We can show that Y is isomorphic to $S/\mathcal{H}$. In other words, a completely 0-simple semigroup S is a rectangular 0-band Y of $\mathcal{H}$-classes, where an $\mathcal{H}$-class H is either a group or $H^2 = 0$. Conversely, suppose S is a rectangular 0-band Y of sets H_α with $\alpha \in Y$. Here H_α is either a group or ${H_\alpha}^2 = 0$. We can show that S is a completely 0-simple semigroup.

Theorem 3.1. *A compleltely 0-simple semigroup $\mathcal{M}^0[G; I, \Lambda; P]$ is 0-tight if and only if the rectangular 0-band $(I \times \Lambda) \cup \{0\}$ whose multiplication is in terms of P with entries in G^0 is 0-tight.*

Proof. Let $S = \mathcal{M}^0[G; I, \Lambda; P]$, and let $Y = (I \times \Lambda) \cup \{0\}$. Suppose that Y is 0-tight. By Theorem 2.2 there are cases of equivalences $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$. We only verify the case where $\mathcal{E}_I = 1_I$ and $\mathcal{E}_\Lambda = 1_\Lambda$. In this case, $\mathcal{S} = 1_I$ and $\mathcal{T} = 1_\Lambda$.

Let ρ be a proper congruence on S. To show that ρ is uniquely determined by each of its congruence classes that do not contain zero, it suffices to verify the following:

$$(i, a, \lambda)\,\rho = \{(i, ga, \lambda) : g \in N\}.$$

If $(i, a, \lambda)\ \rho\ (i, b, \lambda)$, then we deduce that $(p_{\xi i})ab^{-1}(p_{\xi i})^{-1} \in N$ for some $\xi \in \Lambda$ such that $p_{\xi i} \neq 0$ (see p. 88 in [2]). Since N is a normal subgroup of G, we have $ab^{-1} \in N$. It follows that $b = ga$ for $g \in N$.

To show the converse, if $g \in N$, then $(p_{\xi i})(ga)a^{-1}(p_{\xi i})^{-1} = (p_{\xi i})g(p_{\xi i})^{-1}$ in N for every $\xi \in \Lambda$ such that $p_{\xi i} \neq 0$. Hence $(i, a, \lambda)\ \rho\ (i, b, \lambda)$.

Notice that now every congruence on S is uniquely determined by each of the congruence classes that do not contain zero. So S is a 0-tight semigroup. Conversely, suppose S is 0-tight. Since Y is a homomorphic image of S, Y is 0-tight. □

Unlike completely 0-simple semigroups, we do not need equivalences $\mathcal{E}_I$ and $\mathcal{E}_\Lambda$ to investigate congruences on completely simple semigroups. We only have to discuss the equivalence relations $\mathcal{S}$ and $\mathcal{T}$ on I and Λ, respectively (see p. 90 in [2]). This makes our classification a lot easier.

To classify finite tight rectangular bands, we consider various partitions of m and n. There is a specific difference between the classification of finite rectangular 0-bands and that of finite rectangular bands. Let S be a finite rectangular band. We adopt similar terminology in Section 2.

(1) If $|\Lambda| = 1$ and $|I| = 1$ or 2, then S is tight.
(2) If $|\Lambda| = 1$ and $m \geq |I| \geq 3$, then S is tight-free.
(3) If $|\Lambda| = 2$ and $|I| = 2$, then S is tight.
(4) If $|\Lambda| = 2$ and $m \geq |I| \geq 3$, then S is of the third type.
(5) If $|\Lambda| = n \geq 3$ and $m \geq |I| \geq n$, then S is tight-free.

Again there is a duality between finite sets I and Λ. As proved in Theorem 1.1.3 in [2], *a semigroup is a rectangular band if and only if it is isomorphic to the direct product of a left zero semigroup* `A` *and a right zero semigroup* `B`. The proof of Corollary 3.2 follows the proof of Theorem 2.2.

Corollary 3.2. *Let S be a rectangular band. Then the following statements are equivalent:*

(1) *S is tight.*
(2) *1_S is uniquely determined by each of its congruence classes.*
(3) *$|A| \leq 2$ and $|B| \leq 2$.*

We recall that *a completely simple semigroup is a rectangular band of groups, and, conversely, a rectangular band of groups is completely simple* (see p. 80 in [1]). The next corollary follows Theorem 3.1.

Corollary 3.2. *A completely simple semigroup* $\mathcal{M}[G; I, \Lambda; P]$ *is tight if and only if the rectangular band* $I \times \Lambda$ *is tight.*

References

1. A. H. Clifford and G. B. Preston, The algebraic theory of semigroups, Vol. I (1961), Math. Surveys of the Amer. Math. Soc., 7, Providence.

2. J. M. Howie, *Fundamentals of Semigroup Theory*, Clarendon Press, Oxford, 1995.

3. G. Lallement, A note on congruences on Rees matrix semigroups, *Semigroup Forum*, **8**(1974), 89-92.

4. W. D. Munn, Embedding semigroups in congruence-free semigroups, *Semigroup Forum*, **4**(1972), 46-60.

5. G. B. Preston, Congruences on completely 0-simple semigroups, *Proc. London Math. Soc.* (3), **11** (1961), 557-576.

6. D. Rees, On semi-groups, *Math. Proc. Cambridge Philos. Soc.*, **36**(1940), 387-400.

7. B. M. Schein, A remark concerning congruences on [0-] bisimple inverse semigroups, *Semigroup Forum*, **3**(1971), 80-83.

8. Hsing Y. Wu, Tight congruences on semigroups, Doctoral Dissertation, Univ. of Arkansas, Fayetteville, 2004, 11-18.

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