Topological structure of periodic orbits in the integrable four-vortex motion on sphere

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Abstract

An incompressible and inviscid fluid confined in the surface of a sphere is considered. Since the vorticity is a conserved quantity moving with the uid particle, discrete points in which the vorticity is concentrated evolve like material points, called the vortex points. The motion of the N vortex points is described as a Hamiltonian dynamical system, which is referred to as N-vortex problem[1]. The first step in the study of the N-vortex problem is to deal with integrable motions; The three-vortex problem is unconditionally integrable and has already been solved[2, 3]. On the other hand, the four-vortex motion is integrable only when a certain quantity, called the moment of vorticity vector, is zero at the initial moment. This fact was pointed out by Newton[1] and its complete description has recently been given in [4] with a reduction method, from a four-vortex problem to a three-vortex problem proposed by Rott[5].

The integrable four-vortex problem has another significant meaning. The periodic solution of the N vortex points defines a two-dimensional homeomorphism on the sphere with N fixed points, which is called the N – braids. The N-braids is topologically classified into periodic, pseudo – Anosov(pA) and reducible according to Thurston-Nielsen theory[6]. The pA map induces a complex behavior, which is known as topological chaos[7]. Thus much attention has been paid to the chaotic mixing of passive scalars due to the pA periodic orbits of the N-vortex points[8]. Since it is mathematically proved that the 3-braid on sphere is always periodic[9, 10], it is unable to find any periodic orbit that defines the pA braid in the integrable three-vortex
problem. Therefore, we need to consider the $N$-vortex problem for more than three vortex points. On the other hand, while the motion of many vortex points is generally too complicated to find a periodic orbit, the integrable four-vortex problem is better for this purpose, since it provides us with infinitely many periodic orbits. Furthermore, the $pA$ 4-braids is regarded as one of the simplest coherent vortex dynamics that induces the topological chaos on the sphere. In the present talk, based on the analysis in [4], we look for periodic orbits that induce the pseudo-Anosov map on the sphere. We just mention how the topological structure of the periodic orbit relates to the statistical property of the turbulent ow on the sphere.

We also consider whether or not the self-similar collapse is possible. The preceding paper[4] showed no existence of the four-vortex collapse and gave necessary conditions for the self-similar collapse of the partial three vortex points and the pairs of two vortex points. As a result, while the binary collapse is impossible, the triple collapse is unable to be ruled out theoretically. We show numerically that the partial triple collapse is probable.

References