Existence of isoperimetric regions in contact sub-Riemannian manifolds

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Abstract

A contact sub-Riemannian manifold is a contact manifold with a sub-Riemannian metric defined on its horizontal distribution. Isoperimetric inequalities, where the perimeter is the sub-Riemannian one and the volume is Popp’s measure [5], can be considered in such manifolds. Pansu [8] first proved an isoperimetric inequality of the type $|\partial \Omega| \geq C|\Omega|^{4/3}$ in the first Heisenberg group $\mathbb{H}^1$. He also conjectured [7] that equality for the optimal constant is achieved by a distinguished family of spheres with constant mean curvature in $\mathbb{H}^1$. Chanillo and Yang [2] recently extended Pansu’s inequality to pseudo-hermitian 3-manifolds without torsion. A detailed account of recent advances on the isoperimetric inequality in the Heisenberg group $\mathbb{H}^1$ can be found in [1].

Apart from compact sub-Riemannian manifolds, the only known existence result for isoperimetric regions in sub-Riemannian Geometry has been given by Leonardi and Rigot (for Carnot groups) [4].

In this talk I will describe recent joint work with Matteo Galli [3]. We have proven an existence result for isoperimetric regions in contact sub-Riemannian manifolds such that their quotient by the group of contact isometries, the diffeomorphisms that preserve the contact structure and the sub-Riemannian metric, is compact. This is an analog of a useful result in Riemannian Geometry proven by Frank Morgan [6].

References


