Strong edge-coloring on planar graphs

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Abstract

A strong $k$-edge-coloring of a graph $G$ is a mapping from $E(G)$ to $\{1, 2, \ldots, k\}$ such that every two adjacent edges or two edges adjacent to a same edge receive two distinct colors. The strong chromatic index of $G$, denoted by $\chi'_s(G)$, is the smallest integer $k$ such that $G$ admits a strong $k$-edge-coloring.

Strong edge-coloring was first studied by Fouquet and Jolivet (1983, 1984) for cubic planar graphs. A trivial upper bound is that $\chi'_s(G) \leq 2\Delta^2 - 2\Delta + 1$ for any graph $G$ of maximum degree $\Delta$. Fouquet and Jolivet (1983) established a Brooks type upper bound $\chi'_s(G) \leq 2\Delta^2 - 2\Delta$, which is not true only for $G = C_5$ as pointed out by Shiu and Tam (2009). The following conjecture was posed by Erdős and Nešetřil (1988, 1989) and revised by Faudree, Schelp, Gyárfás and Tuza (1990): For any graph $G$ of maximum degree $\Delta$,

$$\chi'_s(G) \leq \frac{5}{4} \Delta^2 \text{ if } \Delta \text{ is even and } \chi'_s(G) \leq \frac{5}{4} \Delta^2 - \frac{1}{2} \Delta + \frac{1}{4} \text{ otherwise.}$$

Faudree, Schelp, Gyárfás and Tuza (1990) also asked whether $\chi'_s(G) \leq 9$ if $G$ is cubic planar. If this upper bound is proved to be true, it would be the best possible. For graph with maximum degree $\Delta = 3$, the above conjecture was verified by Andersen (1992) and by Horák, Qing and Trotter (1993) independently. For $\Delta = 4$, while the conjecture says that $\chi'_s(G) \leq 20$, Horák (1990) obtained $\chi'_s(G) \leq 23$ and Cranston (2006) proved $\chi'_s(G) \leq 22$.

In this talk, we survey recent progress for the strong edge-coloring on planar graphs, including Halin graphs and planar graphs with large girth.