Duality of holomorphic function spaces and smoothing properties of the Bergman projection

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Abstract

Let $\Omega \subset \subset \mathbb{C}^n$ be a domain with smooth boundary, whose Bergman projection $B$ maps the Sobolev space $H^{k_1}(\Omega)$ (continuously) into $H^{k_2}(\Omega)$. We establish two smoothing results: (i) the full Sobolev norm $\|Bf\|_{k_2}$ is controlled by $L^2$ derivatives of $f$ taken along a single, distinguished direction (of order $\leq k_1$), and (ii) the projection of a conjugate holomorphic function in $L^2(\Omega)$ is automatically in $H^{k_2}(\Omega)$. There are obvious corollaries for when $B$ is globally regular.