

On the uniqueness part of Shahidi's L -packet conjecture

Hiraku Atobe

Kyoto University
E-mail: atobe@math.kyoto-u.ac.jp

Abstract

The local Langlands conjecture for quasi-split classical groups G over a non-archimedean local field F has been established by Arthur and Mok. This conjecture says that the set of irreducible tempered smooth representations of $G(F)$ is divided by finite sets, which are called tempered L -packets.

A Whittaker datum is a conjugacy class of a pair $\mathfrak{w} = (B, \mu)$, where $B = TU$ is an F -rational Borel subgroup of G , and μ is a generic character of $U(F)$. An irreducible smooth representation π of $G(F)$ is called generic with respect to \mathfrak{w} if π admits a \mathfrak{w} -Whittaker model. Generic representations are important for both the representation theory and the arithmetic theory.

In 1990, Shahidi predicted that each tempered L -packet has a generic representation with respect to a fixed Whittaker datum for G . One can also consider the uniqueness, i.e., one can expect that such a generic representation would be unique in each L -packet. A proof of the existence part is given in the books of Arthur and Mok. Also, both of the existence and uniqueness parts of Shahidi's conjecture are a special case of the local Gan–Gross–Prasad conjecture, which is now established by Waldspurger, Beuzart-Plessis, Gan–Ichino, and myself. However, these proofs are long and complicated after the works of Arthur and Mok.

In this talk, I will give another proof of the uniqueness part of Shahidi's conjecture. In conclusion, Shahidi's conjecture is a formal consequence of the works of Arthur and Mok.

Comparison of local spherical characters and the Ichino-Ikeda conjecture for unitary groups

Raphael Beuzart-Plessis

National University of Singapore
E-mail: matbrf@nus.edu.sg

Abstract

The global Gan-Gross-Prasad conjectures relate the non-vanishing of certain automorphic periods to the non-vanishing of central special values of L -functions. These conjectures have been refined by Ichino-Ikeda and N.Harris into exact formulas involving the relevant L -functions and the square of the absolute value of the corresponding periods. In his seminal work, Wei Zhang has proved both conjectures for cuspidal automorphic representations of unitary groups satisfying certain local conditions. These local conditions are actually far more stringent for the Ichino-Ikeda conjecture than for the Gan-Gross-Prasad conjecture and Zhang has stated a local conjecture which, if true, would allow to relax most of these restrictions. In this talk, I shall present a proof of this conjecture for all p -adic fields. The main tools of the proof are certain local analogues of the so-called Jacquet-Rallis trace formulas, which have played a prominent role in the work of Zhang, together with some "truncated" local expansions of relative characters which have been obtained previously by Zhang.

Nonstandard Fourier transforms on $\mathrm{GL}(n)$

Shuyang Cheng

University of Chicago
E-mail: shu@math.uchicago.edu

Abstract

Analytic properties of standard L -functions $L(s, \pi)$ on $\mathrm{GL}(n)$ such as analytic continuation and functional equation have been established by studying local and global Fourier transforms on the vector space of $n \times n$ matrices by Iwasawa–Tate for $n = 1$ and Godement–Jacquet for general n . Conjecturally after the work of Braverman–Kazhdan, Lafforgue and Ngô, there exist nonstandard Fourier transforms which are related to automorphic L -functions $L(s, \pi, \rho)$ for nonstandard ρ in a similar manner. In my talk I plan to give an introduction to this circle of ideas together with analogues over finite fields and, if time permits, sketch a proof of an acyclicity conjecture of Braverman–Kazhdan for $\mathrm{GL}(n)$. This is joint work with B. C. Ngô.

Fourier Jacobi coefficients of Siegel modular forms

Soumya Das

Indian Institute of Science
E-mail: somu@math.iisc.ernet.in

Abstract

Fourier Jacobi expansions of Siegel modular forms is an useful tool and has many applications. In this talk I would discuss these objects both for cusp forms and Eisenstein series, and about Petersson inner products of such coefficients. Some applications to representation numbers of quadratic forms would be discussed. This is part of a joint work with S. Böcherer.

Applications of the Deligne-Kazhdan philosophy to the Langlands correspondence for split classical groups

Radhika Ganapathy, Sandeep Varma

Tata Institute of Fundamental Research
E-mail:radhika@math.tifr.res.in

Abstract

The Deligne-Kazhdan theory loosely says that the complex representation theory of Galois groups and split reductive groups over a local field of characteristic p can be viewed as the limit, as the ramification index tends to infinity, of the representation theory of these groups over local fields of characteristic 0. In this talk, I will explain how this method can be used to prove the local Langlands correspondence for split classical groups over a local field of characteristic p (with some restrictions on the characteristic) using the corresponding result of Arthur in characteristic 0. This is joint work with Sandeep Varma.

On signs of cusp forms and the halting of an algorithm to construct a supersingular elliptic curve with a given endomorphism ring

Ben Kane and King Cheong (Lawrence) Fung

University of Hong Kong
E-mail: bkane@hku.hk

Abstract

In this talk, we will consider an application coming from the alternation of signs of Fourier coefficients of (half-integral weight) cusp forms. In particular, we consider certain cusp forms coming from the difference of two ternary theta functions associated to the norm map applied to trace zero elements within a maximal order of the definite quaternion algebra ramified precisely at p and ∞ . It was conjectured by Cheyreve and Galbraith that one such theta function could not “dominate” the other, i.e., its Fourier coefficients essentially could not always be larger. We prove this conjecture by recognizing that the coefficients of the difference change sign infinitely often due to recent work of others on sign changes. The conjecture of Cheyreve and Galbraith was originally made because it implies that a certain algorithm they had developed would halt with the correct answer, and hence as a corollary we conclude that their algorithm indeed halts. This is joint work with my Masters student King Cheong Fung, who is investigating further related directions.

Pan Asia Number Theory Conference 2016
Institute of Mathematics, Academia Sinica, Taipei, Taiwan
July 11 – 15, 2016

Heights of motives

Teruhisa Koshikawa

RIMS, Kyoto University
E-mail: teruhisa1210@hotmail.com

Abstract

Kazuya Kato proposed “height” of a pure motive over a global field generalizing the Faltings height of an abelian variety. I will explain a modified version of the definition of the height and how to generalize some properties of Faltings height.

On Full BSD Conjecture for Rank One CM Elliptic Curves

Yongxiong Li, Yu Liu and Ye Tian

Tsinghua University
E-mail: *yongxiongli@math.tsinghua.edu.cn*

Abstract

Let E be an elliptic curve defined over \mathbb{Q} with complex multiplication by K . Suppose that E has analytic rank one. For odd potential good ordinary primes p of E , we show the p -part full BSD for E . It is known by Perrin-Riou and Kobayashi for all good odd primes p .

The talk is based on the joint work with Yu Liu and Ye Tian.

Crystalline lift and overconvergence of p-adic Galois representations

Tong Liu

Purdue University
E-mail: tongliu@purdue.edu

Abstract

We prove the overconvergence of p-adic Galois representation in the Breuil-Kisin setting (i.e., the case that K_∞ is a Kummer extension). We first show that each p^n -torsion representation is a quotient of two lattices in a crystalline representation. This allows us to control the height (in terms of n) of (maximal) Kisin model in each p^n -torsion representation. It turns out that the height of such Kisin model strongly relates to the overconvergent radius and then the overconvergence of p-adic Galois representation follows. This is collaborated work with Hui Gao.

Fourier coefficients of automorphic forms for higher rank groups

Guangshi Lü

Shandong University
E-mail: gslv@sdu.edu.cn

Abstract

In this talk, I shall introduce some results on the distribution of Fourier coefficients of automorphic forms for higher rank groups. In particular, I will talk about the orthogonality between additive characters and Fourier coefficients over primes, and Bombieri-Vinogradov type mean value connected with Fourier coefficients. These results are related to the Möbius Randomness Principle.

A construction of the local epsilon isomorphisms using Colmez's multiplicative convolution

Kentaro Nakamura

Saga University
E-mail: nkentaro@cc.saga-u.ac.jp

Abstract

In my talk, I will explain a conjecture concerning the construction of the local epsilon isomorphisms for arbitrary families of p-adic representations of $\text{Gal}(\bar{Q}_p/Q_p)$ by using Colmez's multiplicative convolution defined for (ϕ, Γ) -modules.

On mod p local-global compatibility for $\mathrm{GL}_3(\mathbb{Q}_p)$ in the non-ordinary case

Chol Park

Korea Institute for Advanced Study
E-mail: cpark@kias.re.kr

Abstract

Let F/\mathbb{Q} be a CM field in which p splits completely and $\bar{r} : \mathrm{Gal}(\bar{\mathbb{Q}}/F) \rightarrow \mathrm{GL}_3(\bar{\mathbb{F}}_p)$ a continuous modular Galois representation. We assume that $\bar{r}|_{\mathrm{Gal}(\bar{\mathbb{Q}}_p/F_w)}$ is a non-trivial extension of two dimensional irreducible representation by a character at a place w above p . In this talk, we discuss a problem about local-global compatibility in the mod p Langlands program for $\mathrm{GL}_3(\mathbb{Q}_p)$. We define a local invariant associated to $\bar{r}|_{\mathrm{Gal}(\bar{\mathbb{Q}}_p/F_w)}$ in terms of Fontaine-Laffaille theory and provide a nearly optimal weight elimination result as well as the modularity of the obvious weights of \bar{r} . We show that the local invariant associated to $\bar{r}|_{\mathrm{Gal}(\bar{\mathbb{Q}}_p/F_w)}$ can be obtained in terms of a refined Hecke action on a space of mod p algebraic automorphic forms on a compact unitary group.

This is a joint work with Daniel Le and Stefano Morra.

Characteristic cycle of a constructible sheaf

Takeshi Saito

University of Tokyo
E-mail: t-saito@ms.u-tokyo.ac.jp

Abstract

The characteristic cycle of a constructible sheaf is a linear combination of irreducible components of the singular support defined recently by Beilinson. I will explain some basic examples and the functoriality including the index formula that is a generalization of the Grothendieck-Ogg-Shafarevich formula to higher dimension.

On construction of higher rank Kolyvagin systems

Takamichi Sano

Osaka University
E-mail: tkmc310@gmail.com

Abstract

Recently, Mazur and Rubin initiated the theory of higher rank Kolyvagin systems. Although it is expected that higher rank Kolyvagin systems are constructed from higher rank Euler systems, no general construction is known except for the rank one case. In this talk, I will explain that a higher rank Euler system is constructed from elements in determinants of étale cohomology (in an algebraic way), and that a higher rank Kolyvagin system is naturally constructed from such an Euler system. If time permits, I will also talk about several results on higher Fitting ideals of Selmer groups. This is joint work with David Burns.

The arithmetic of modular symbols

Romyar Sharifi

The University of Arizona
E-mail: sharifi@math.arizona.edu

Abstract

I will describe an explicit conjectural relationship between modular symbols in the homology of modular curves modulo Eisenstein ideals and products of cyclotomic units in the cohomology of cyclotomic integer rings. I intend to survey results and work-in-progress on this conjecture and its refinements, including the development of higher-dimensional analogues over certain global fields that is joint work with Takako Fukaya and Kazuya Kato.

The Iwasawa Main Conjecture at non-ordinary primes

Florian Sprung

Princeton University/IAS
E-mail: fsprung@princeton.edu

Abstract

Given an elliptic curve E , the Birch and Swinnerton-Dyer conjecture predicts not only the correct order of vanishing of the L -function of E at 1, but also the leading Taylor coefficient of this L -function in terms of algebraic invariants. One strategy for proving this is to do this prime by prime, by proving an appropriate Iwasawa main conjecture at every prime p . For good ordinary p , the main conjecture follows from work of Rubin (the CM case), Kato and Skinner–Urban. For non-ordinary primes, the result is known via works of Pollack–Rubin (again in the CM case, in which $a_p = 0$), and Kato/Kobayashi for one inclusion. Here, a_p is the error term in counting $\#E(\mathbb{F}_p)$. The other inclusion has been handled by Wan when $a_p = 0$. We present a strategy for a proof that includes the case $a_p \neq 0$ as well.