

Twisted Bhargava cubes and boxes

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Abstract

In his groundbreaking thesis work from 2001, Manjul Bhargava extended Gauss's composition laws for binary quadratic forms to higher degree forms. One crucial ingredient in his work is the parametrisation of the orbits of lattice points in a prehomogeneous vector space by quantities of arithmetic interest. Using the $SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z})$ -action on $2 \times 2 \times 2$ cubes, he gave a simpler description of Gauss's law, and using the $SL(2, \mathbb{Z}) \times SL(3, \mathbb{Z}) \times SL(3, \mathbb{Z})$ -action on $2 \times 3 \times 3$ boxes, he discovered a composition law on binary cubic forms. In this lecture, we revisit these two orbit problems over fields and consider twisted versions of them. In particular, we shall see that the orbits in both problems are parametrised by the same class of objects.