

# On signs of cusp forms and the halting of an algorithm to construct a supersingular elliptic curve with a given endomorphism ring

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## Abstract

In this talk, we will consider an application coming from the alternation of signs of Fourier coefficients of (half-integral weight) cusp forms. In particular, we consider certain cusp forms coming from the difference of two ternary theta functions associated to the norm map applied to trace zero elements within a maximal order of the definite quaternion algebra ramified precisely at  $p$  and  $\infty$ . It was conjectured by Cheyreve and Galbraith that one such theta function could not “dominate” the other, i.e., its Fourier coefficients essentially could not always be larger. We prove this conjecture by recognizing that the coefficients of the difference change sign infinitely often due to recent work of others on sign changes. The conjecture of Cheyreve and Galbraith was originally made because it implies that a certain algorithm they had developed would halt with the correct answer, and hence as a corollary we conclude that their algorithm indeed halts. This is joint work with my Masters student King Cheong Fung, who is investigating further related directions.