

# The Iwasawa Main Conjecture at non-ordinary primes

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## Abstract

Given an elliptic curve  $E$ , the Birch and Swinnerton-Dyer conjecture predicts not only the correct order of vanishing of the  $L$ -function of  $E$  at 1, but also the leading Taylor coefficient of this  $L$ -function in terms of algebraic invariants. One strategy for proving this is to do this prime by prime, by proving an appropriate Iwasawa main conjecture at every prime  $p$ . For good ordinary  $p$ , the main conjecture follows from work of Rubin (the CM case), Kato and Skinner–Urban. For non-ordinary primes, the result is known via works of Pollack–Rubin (again in the CM case, in which  $a_p = 0$ ), and Kato/Kobayashi for one inclusion. Here,  $a_p$  is the error term in counting  $\#E(\mathbb{F}_p)$ . The other inclusion has been handled by Wan when  $a_p = 0$ . We present a strategy for a proof that includes the case  $a_p \neq 0$  as well.