

中央研究院數學研究所

Institute of Mathematics, Academia Sinica

Taipei Postdoc Seminar

Speaker : 朴振完 Jinwan Park (本所 Academia Sinica)

Title: The Regularity Theory for the Double Obstacle Problem

Abstract :

In this talk, I will introduce the regularity of the free boundary for the double obstacle problems. Precisely, I will discuss the interior C^1 regularity of the free boundary for the elliptic and parabolic double obstacle for linear and Fully nonlinear operator. Furthermore, I will introduce the C^1 regularity of the free boundary near fixed boundary for Laplacian problem.

First, I am going to introduce the proof of local C^1 regularity of free boundaries for the double obstacle problem with an upper obstacle ψ ,

$$\Delta u = f\chi_{\Omega(u)\cap\{u<\psi\}} + \Delta\psi\chi_{\Omega(u)\cap\{u=\psi\}}, \quad u \leq \psi \quad \text{in } B_1,$$

where $\Omega(u) = B_1 \setminus (\{u = 0\} \cap \{\nabla u = 0\})$ under a thickness assumption for u and ψ . The function ψ satisfies

$$\psi \in C^{1,1}(B_1) \cap C^{2,1}(\overline{\Omega(\psi)}), \quad \Omega(\psi) = B_1 \setminus (\{\psi = 0\} \cap \{\nabla\psi = 0\}).$$

Our result is very close to the well-known regularity theory of L. Caffarelli for the obstacle problem [Caf77], and also the no-sign obstacle problem due to Caffarelli-Karp-Shahgholian [CKS00]. This is a joint work with Ki-ahm Lee and Henrik Shahgholian.

Next, I will talk about the problem for elliptic fully nonlinear operator. Because of the non-linearity of the operator, the main difficulty in our work is the lack of monotonicity formulas, which is the important tools for the classification of the global solutions for the double obstacle problem for the Laplace operator. Thus, we exploit the method that use the value $\partial_e u/x_n$, where u is the global solution with the upper obstacle $\psi = c(x_n^+)^2$. The method using the first derivative first introduced in [IM16], in the study of the regularity of the free boundary near fixed boundary for single fully nonlinear obstacle problem.

Lastly, I will briefly introduce the result on parabolic double obstacle problem and the C^1 regularity of the free boundary near fixed boundary for Laplacian problem.

BIBLIOGRAPHY

- [Caf77] Luis A Caffarelli. The regularity of free boundaries in higher dimensions. *Acta Mathematica*, 139:155–184, 1977.
- [CKS00] Luis A Caffarelli, Lavi Karp, and Henrik Shahgholian. Regularity of a free boundary with application to the pompeiu problem. *Annals of Mathematics*, 151(1):269–292, 2000.
- [IM16] Emanuel Indrei and Andreas Minne. Non-transversal intersection of free and fixed boundary for fully nonlinear elliptic operators in two dimensions. *Analysis and PDE*, 9(2):487–502, 2016.

Time : 11:00 - 12:30, Wednesday, April 15, 2020

Venue : Online (TBA)

Organizer : Sheng-Fu Chiu (Academia Sinica), Jia-Yuan Dai (National Center for Theoretical Sciences)

https://www.math.sinica.edu.tw/www/file_upload/conference/2016TPS/index.html

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