

中央研究院數學研究所

Institute of Mathematics, Academia Sinica

Taipei Postdoc Seminar

Speaker : 鄭堯 Yao Cheng (本所 Academia Sinica)

Title : On the newform problem for split SO_{2n+1}

Abstract :

Let $p > 0$ be a prime integer and denote \mathbb{Q}_p to be the field of topological completion of \mathbb{Q} under the p -adic topology. The newform problem for $GL_2(\mathbb{Q}_p)$ concerns the following questions. Let $m \geq 0$ be an integer and define the subgroup $\Gamma_{2,m}$ of $GL_2(\mathbb{Z}_p)$ by

$$\Gamma_{2,m} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(\mathbb{Z}_p) \mid c, d-1 \in p^m \mathbb{Z}_p \right\}.$$

Let π be an irreducible smooth infinite-dimensional (complex) representation of $GL_2(\mathbb{Q}_p)$ with underlying \mathbb{C} -linear space \mathcal{V}_π . Consider the sequence

$$\mathcal{V}_\pi^{\Gamma_{2,0}}, \mathcal{V}_\pi^{\Gamma_{2,1}}, \dots, \mathcal{V}_\pi^{\Gamma_{2,m}}, \dots$$

of subspaces consisting of $\Gamma_{2,m}$ -fixed elements in \mathcal{V}_π for $m = 0, 1, 2, \dots$. A basic result in the representation theory implies that every subspaces appearing in the sequence are finite-dimensional. Now we ask:

- (a) are $\mathcal{V}_\pi^{\Gamma_{2,m}} = \{0\}$ for all m ?
- (b) if the answer to question (a) is negative, then can we determine the smallest m so that $\mathcal{V}_\pi^{\Gamma_{2,m}} \neq \{0\}$, and moreover, can we determine $\dim_{\mathbb{C}} \mathcal{V}_\pi^{\Gamma_{2,m}}$ for such m ?

Questions (a) and (b) are completely addressed by W. Casselman in 1973. In view of the isomorphisms $PGL_3 \cong SO_3$, $PGSp_4 \cong SO_5$ and the results of Roberts-Schmidt in 2007, B. Gross and P-Y. Tsai defined a family $\{K_{n,m}\}_{m \geq 0}$ of subgroups in $SO_{2n+1}(\mathbb{Q}_p)$ which are essentially $\{\Gamma_{2,m}\}_{m \geq 0}$ when $n = 1$. Let π be an irreducible smooth generic representation of $SO_{2n+1}(\mathbb{Q}_p)$. They then asked the same questions for π but now with respect to $\{K_{n,m}\}_{m \geq 0}$. In her thesis, P-Y Tsai proved that the answer to question (a) is negative and they proposed conjectural answers to question (b). Furthermore, P-Y Tsai treated this conjecture for the generic supercuspidal representations in her thesis. In this talk, we will introduce their conjecture and state our current results toward this conjecture for general generic representations with sketchy proofs (if possible).

Time : 13:30 - 15:00, Wednesday, October 7, 2020

Venue : Lecture Hall 5F, Cosmology Building (NTU Campus) 次震宇宙館 五樓演講廳

Organizer : Wei-Bo Su (Academia Sinica), Peng-Jie Wong (National Center for Theoretical Sciences)

https://www.math.sinica.edu.tw/www/file_upload/conference/2016TPS/index.html

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