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Context-free Grammars for Combinatorial Enumeration

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Abstract

A context-free grammar is a set of production rules over an alphabet or a set of variables. It can be viewed as a creation operator acting on multivariate Laurent polynomials. On one hand, a grammar may be used to capture certain features in terms of grammatical labelings in the generation of a combinatorial structure. It can be also useful in constructing bijections. On the other hand, a grammar can be directly employed in a rigorous calculus in the spirit of the classical symbolic method. This approach fits in a great many combinatorial objects, and so far it remains the only means to tackle some enumeration and positivity problems. Most noticeably, Dumont discovered the grammars for the Eulerian polynomials, up-down permutations and rooted trees. A slight extension of Dumont's grammar gives rise to a grammar for the second order Eulerian polynomials on Stirling permutations. Intuitively, the merit of using grammars lies in the freedom of making use of many variables, whereas the corresponding operator still enjoys the property of a derivative. In this talk, we will discuss the basic ideas and will report on recent developments on combinatorial polynomials, permutation statistics, γ-positivity (partial γ-positivity and bi-γ-positivity), stable polynomials, trees and tableaux, due to W.Y.C. Chen, A.M. Fu, X.J. Hao, G.N. Han, F.Z.K. Li, Z.C. Lin, J. Ma, S.M. Ma, T. Mansour, M.A. Mendez, J.L. Ramirez, H.R.L. Yang, J. Yeh, Y.N. Yeh, B.X. Zhu and their collaborators.