One-Day Workshop on Integrable Systems

Sunday, December 17, 2023

Institute of Mathematics, Academia Sinica

Taipei City, Taiwan

Introduction

The study of physical phenomena via mathematical models often leads to integrable systems which admit rich solutions, new concepts, and techniques. This mini workshop is aimed to bring together a group of scientists to discuss various aspects, methods, and problems in integrable systems.

Venue

Room 617, 6F, Astronomy-Mathematics Building (NTU campus), No. 1, Sec. 4, Roosevelt Road, Taipei City

Speakers

- Kuo, Ting-Jung (National Taiwan Normal University)
- Chang, Jen-Hsu (National Defense University)
- Wu, Derchyi (Academia Sinica, Taipei)
- Lin, Chang-Shou (National Taiwan University)
- Miller, Peter (University of Michigan, Ann Arbor)
- Law, Chun-Kong (National Sun Yat-sen University)

Agenda

9:00-9:45 Kuo, Ting-Jung

Title: A new isomonodromy deformation equation with movable branch points

Abstract: The isomonodromic deformation plays a universal role in connecting various research areas of mathematics and physics. In this talk, I am going to discuss the isomonodromy theory for a new class of Fuchsian-type elliptic second-order equations defined on the moduli space of elliptic curves with the parameter $\tau \in \mathbf{H}$ (upper half-plane). We will observe that the isomonodromic deformation equation is governed by a new second-order nonlinear equation with a deep connection to the Painlevé VI equation but admits essentially different properties from PVI. Indeed, the new isomonodromic deformation equation, distinct from the Painlevé VI equation, admits so-called movable branch points that can be explicitly determined by their monodromy data.

10:00-10:45 Chang, Jen-Hsu

Title: Real multi-line solitons of the BKP equation

Abstract: The BKP equation or the 2+1 Sawada-Kotera equation

$$(9\phi_t - 5\phi_{xxy} + \phi_{xxxxx} - 15\phi_x\phi_y + 15\phi_x\phi_{xxx} + 15\phi_x^3)_x - 5\phi_{yy} = 0$$

is obtained from the reduction of B-type in the KP hierarchy under the orthogonal type transformation group for the KP equation. It can also be obtained by the Kupershmidt reduction. Its multi-line soliton solutions of BKP equation can be constructed by the Pfaffian structure. Then one obtains the resonant structure of real line solitons of BKP equation using the totally non-negative Grassmannian in the Pfaffian structure. Especially, the N-solitons solution is studied and its self-dual Tau function is obtained. Also, one can construct the totally non-negative Grassmannian of the (1+1)-Sawada-Kotera equation, that is, $\phi_y = 0$,

$$(9\phi_t + \phi_{xxxxx} + 15\phi_x\phi_{xxx} + 15\phi_x^3) = 0,$$

for its real line solitons.

11:00-11:45 Wu, Derchyi

Title: Stability of KdV n-solitons

Abstract: The Korteweg-de Vries equation

$$u_t - 6uu_x + u_{xxx} = 0$$

is an asymptotic model for the propagation of one-dimensional small amplitude long surface waves. Special solitary solutions

$$u(x,t) = -2\beta^2 \mathrm{sech}^2(\beta x - 4\beta^3 t + \frac{1}{2}\ln\alpha)$$

are the simplest examples in an important class of KdV solutions, called the KdV n-solitons. They represent non linear superpositions of n travelling waves with different speeds β_j which interact and remain unchanged after interaction, and behave asymptotically as the sum of n travelling waves as $t \to +\infty$. Via PDE approaches, orbital stability theorems of KdV n-solitons in various Sobolev spaces have been justified since 1975. Integrability is hardly or less exploited in these PDE approaches. Hence the methods work for non integrable systems with low regularities but detailed description of the evolution profile is missing.

Applying the inverse scattering theory, we present an orbital stability theorem of KdV n-solitons with explicit phase shifts.

14:00-14:45 Lin, Chang-Shou

Title: Some program on second order Fuchsian ODEs with elliptic coefficients

Abstract: I will present my recent study on ODEs with complex variables. The purpose of this study is to generalize the theory of elliptic KdV potentials from the aspects of monodromy data.

15:00-15:45 Miller, Peter

Title: Soliton ensembles for the Benjamin-Ono equation with small dispersion: a tale of two matrices

Abstract: A soliton ensemble is an exact multi-soliton solution of a given integrable equation that is meant to approximate the solution of the Cauchy problem with specified initial data. The approximation is accurate when the dispersion coefficient is small, which also makes the number of solitons large. We explain this in the setting of the Benjamin-Ono equation, and show how the soliton ensemble can be expressed explicitly in terms of the eigenvalues of two different types of large matrices, one Hermitian and depending on both space and time variables, and one non-Hermitian but depending only on time. We illustrate the distribution of the eigenvalues of each matrix in the small-dispersion limit and formulate corresponding conjectures based on numerical experiments. The conjectures imply strong asymptotics accurately resolving rapid oscillations for the soliton ensemble even beyond the formation time of a dispersive shock wave. This is joint work with Elliot Blackstone and Louise Gassot.

16:00-16:45 Law, Chun-Kong

Title: McCoy-Tracy-Wu solutions for the third Painlevé equation

Abstract: It is now known that the six Painlevé equations (P1-P6) have deep connections with different fields in mathematics and science. Thus their study has been an indispensable part of integrable systems. We prove that there exists a family of bounded solutions of a special case of the third Painlevé equation (P3) with parameters $\alpha = -\beta \in \mathbb{R}$ and $\gamma = -\delta = -1$, having certain asymptotics at $z = 0+$ and $z = \infty$. These solutions were first discovered by Wu-McCoy-Tracy-Barouch to appear in the scaling limit of the spin-spin correlation function of the two-dimensional Ising model. In 1977, McCoy-Tracy- Wu gave a rigorous proof of this existence result by expressing the solution as an infinite series of multiple integrals. We plan to give another proof by shooting argument, as Hastings-McLeod [4] had done for the second Painlevé equation.