New implementation of elliptic systems method for time dependent diffusion tomography with novel boundary treatment for back reflected and transmitted data

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Abstract

It is common in applied work in engineering such as the medical imaging for diagnosis of possible breast tumors, to have only limited boundary measurement information using either back reflected or transmitted data. Here we formulate the problem as one of coefficient recovery from incomplete boundary data in inverse problems. We have completed a new implementation of the Elliptic Systems Method (ESM) in time dependent diffusion tomography. The basic formulation of the ESM involves solving a system of (typically 4) coupled 4th-order partial differential equations, with the time variable integrated out using Legendre polynomials. Here, unlike the previous implementation that creates a larger (typically of size 8) mixed system of 2nd-order problems with quadratic elements over triangles, we use C¹ Bogner-Fox-Schmit bi-cubic elements over rectangles, with a new treatment of boundary conditions in the common case of incomplete boundary data. This new method is 4th-order accurate for sufficiently smooth functions. The new boundary condition approach allows the use of homogeneous natural boundary conditions on parts of the boundary where no measured data is available. This combined effort is being reported elsewhere, but without extensive comparisons of difficult applications against the literature. Here we will focus on three previously published examples using back reflected or transmitted data with one or two inclusions. The new implementation in comparison gives markedly improved results for inclusion recovery, all of which are achieved without use of additional aids such as weight functions which have previously been thought to be essential. In addition the new implementation is shown to be surprisingly robust with respect to noise. We conclude with two examples illustrating the effect of increasing levels of noise.

References

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