# ENDOGENOUS SPECIALISATION AND ENDOGENOUS PRINCIPAL-AGENT RELATIONSHIP*

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The paper develops a general equilibrium model with endogenous principal-agent relationship within a framework of consumer-producer, economies of specialisation, and transaction costs. It is shown that if transaction efficiency is low, then autarky is chosen as the general equilibrium where no market and principal-agent relationship exists. As transaction efficiency is improved, the equilibrium level of division of labour increases, comparative advantage between *ex ante* identical individuals emerges from the division of labour, and the number of principal-agent relationships increases. The following features of the model distinguish it from other principal-agent models in the literature. The principal-agent relationships are not only endogenous, but also reciprocal between different specialists. In a general equilibrium environment, choice between pure pricing and contingent pricing is endogenised. In the paper, the implications of endogenous transaction costs caused by moral hazard for the equilibrium extent of the market and related degrees of market integration, production concentration, trade dependence, diversity of economic structure, and productivity are explored. The model predicts two interesting phenomena: a man might work harder for the market with moral hazard than working for himself in the absence of moral hazard; a market with moral hazard might be Pareto superior to autarky with no moral hazard.

## I. Introduction

The purpose of the paper is to endogenise the principal-agent relationship and transaction costs in a general equilibrium model with consumer-producers, economies of specialisation, and transaction costs. Two phenomena that cannot be explained by existing models of principal-agent motivate this research. From our daily experience, we can see an interesting phenomenon that when an individual works for himself in the absence of moral hazard, he may be not as diligent as working for the other in the market with moral hazard. Why, under a certain condition, does an individual choose a higher effort level when moral hazard is present than when moral hazard is absent? If specialisation and transaction costs are simultaneously endogenised in a general equilibrium model, we can then predict this phenomenon as follows. In the present paper we call a departure of equilibrium from the Pareto optimum endogenous transaction cost and refer to exogenous transaction costs as a kind of transaction costs that can be identified before individuals have made decisions. Endogenous transaction cost cannot be identified before individuals have made decisions and the economy has settled down in

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equilibrium. The trade offs between positive network effect of division of labour, endogenous transaction costs caused by moral hazard, and exogenous transaction costs imply that interactions between two types of transaction costs are crucial determinants of the equilibrium level of division of labour. If an exogenous transaction cost coefficient for each unit of goods traded is very large, the transaction cost outweighs productivity gains from the division of labour. Hence, individuals choose autarky where there is no transaction and related exogenous and endogenous transaction costs. Because of each person's limited time, each person has a narrow scope for trading off positive contribution of effort in reducing low productivity risk against leisure in autarky. Thus, the efficient trade off between the positive contribution and the disutility of the effort may end up with a low effort level in autarky. As the transaction cost coefficient falls, individuals will choose a higher level of division of labour, which implies a higher aggregate productivity, so that the scope for trading off the positive contribution against disutility of risk avoiding effort is enlarged by the higher level of specialisation. This is because each individual has more time for both leisure and working in the activity that he chooses when he increases his level of specialisation (reducing the number of production activities that he undertakes). Also, the decrease in the exogenous transaction cost coefficient enlarges the scope for trading off economies of specialisation against moral hazard. Hence, he may choose a higher level of effort in reducing risk when he chooses to work for others in a larger network of division of labour where endogenous transaction costs are present due to moral hazard. In other words, individuals can afford a higher endogenous transaction cost when aggregate productivity increases as a result of a larger network of division of labour. We may refer to the phenomenon as that a man works harder for others in the presence of moral hazard than working for himself in the absence of moral hazard.

The second phenomenon that motivates the paper relates to the first phenomenon. We can see that in a large and very commercialised city (or with a very sophisticated network of division of labour), a greater interdependence between individuals and a larger number of transactions create more scope for endogenous transaction costs caused by moral hazard and adverse selection than in a remote countryside village in a poor country where the number of transactions is small and no much scope for moral hazard to take place. As a general equilibrium phenomenon, we may observe that residents in that large city are more opportunistic than village folks. But yet people in the city are Pareto better off than those in the poor country. In other words, an equilibrium with moral hazard, which is not Pareto optimal, may be Pareto superior to an equilibrium with no moral hazard under different exogenous transaction conditions. This phenomenon may be referred to as Pareto improvement associated with an increase in moral hazard.

This interesting phenomenon can be predicted by our general equilibrium model with endogenous level of division of labour and the trade offs between endogenous comparative advantage and endogenous and exogenous transaction costs. If the exogenous transaction cost coefficient for each unit of good traded is large, then exogenous transaction cost outweighs endogenous comparative advantage, so that the Pareto optimum as well as the equilibrium is autarky where there is no transactions and moral hazard. As the exogenous transaction cost coefficient falls the scope for trading off endogenous comparative advantage against transaction costs is enlarged, so that the Pareto optimum shifts to a higher level of division of labour which involves more interactions and transactions between individuals. The transactions must involve endogenous transaction cost caused by moral hazard, so that the equilibrium is not Pareto optimal. But as long as endogenous comparative advantage that can be exploited is sufficiently large compared to endogenous and exogenous transaction cost such that individuals can afford the moral hazard caused by the division of labour, the division of labour with more moral hazard occurs at general equilibrium and is Pareto superior to autarky. Hence, productivity, welfare, and moral hazard are simultaneously increased by the fall of the exogenous transaction cost coefficient.

In the existing literature of moral hazard, the principal-agent relationship is exogenously given.[1] The principal cannot take care of his own business and he has to ask an agent to do the job, while the agent cannot have his own business and he has to work for others. Since individuals' levels of specialisation and related principal-agent relationships are not endogenised in this literature, the implications of the endogenous transaction costs caused by moral hazard for the equilibrium level of division of labour are not explored.[2] Hence, the above two phenomena cannot be predicted by the existing models of principal-agent.

The literature of endogenous specialisation has endogenised individuals' levels of specialisation and the level of division of labour for society as a whole on the basis of the trade off between economies of specialisation and exogenous transaction costs. But it has not endogenised transaction costs.[3] Hence, this literature cannot explore the implications of endogenous transaction costs for the level of division of labour and related number of principal-agent relationships either. The current paper introduces moral hazard into the general equilibrium model of endogenous specialisation and develops a general equilibrium principal-agent model with endogenous specialisation to explore the implications of endogenous transaction costs caused by moral hazard for the equilibrium level of division of labour.

The concept of division of labour is in essence a general equilibrium concept. Hence, an individual's level of specialisation and level of division of labour can be endogenised only in a general equilibrium framework. The equilibrium level of division of labour is dependent on endogenous transaction costs (for instance an individual will not choose specialisation if it involves too high endogenous transaction cost), while the endogenous transaction costs are determined by the level of division of labour (for instance endogenous transaction cost does not occur if all individuals choose autarky where no transaction takes place). Hence, a general equilibrium mechanism that simultaneously determines the interdependent variables should be used to explore the implication of endogenous transaction costs for the equilibrium network size of division of labour. But in the existing literature, endogenous transaction costs are investigated for a given pattern of division of labour between the principal and the agent. As shown in the literature of endogenous specialisation, the extent of the market, trade dependence, the degree of market integration, production concentration, the degree of diversification of economic structure, the number of markets, the degree of endogenous comparative advantage, the number of traded goods, the degree of interpersonal dependencies, productivity, and individuals' levels of specialisation are associated with the level of division

[1] The surveys of this literature can be found from Hart and Holmstrom (1987), Arrow (1985), Holmstrom and Roberts (1998), Bolton and Scharfstein (1998), and Gibbons (1998). The tradeoff between exogenous monitoring cost and endogenous transaction costs caused by moral hazard is considered by Holmstrom and Milgrom (1991) and Cowen and Glazer (1996). Implications of residual rights in the model with two sided moral hazard is considered by Gupta and Romano (1998) and Hart (1995), the tradeoff between insurance and exogenous and endogenous transaction costs is considered by Milgrom and Roberts (1992) and others. Unemployment insurance in the principal-agent relationship is considered by Hopenhayn and Nicolini (1997). Laffont and Tirole (1986) and Lewis and Sappington (1991) develop models with hidden information (rather than hidden actions) to endogenise principal-agent relationship by formulating the trade off between exogenous comparative advantage and endogenous transaction costs caused by information asymmetry. Moral hazard in a general equilibrium model is considered in Helpman and Laffont (1975), Kihlstrom and Laffont (1979), and Legros and Newman (1996).

[2] One exception is Legros and Newman (1996) which allows players to choose between agent and principal, though it does not endogenise emergence of the principal-agent relationships.

[3] Surveys of this literature can be found from Yang and Ng (1998) and Yang (2001).

of labour which is determined by the efficient trade off between economies of specialisation and transaction costs. Also, it is shown that emergence of money, business cycles, and unemployment can be explained by evolution in the level of division of labour. In a general equilibrium model with endogenous comparative advantage and exogenous and endogenous transaction costs, complicated trade offs among the three counteracting forces may generate much richer stories than told by the existing literature of principal-agent and by the existing literature of endogenous specialisation.

Yang (1994) has drawn the distinction between endogenous and exogenous comparative advantage. The endogenous comparative advantage is positive network effect of division of labour that can be exploited only if different individuals, who might be *ex ante* identical in all aspects, choose different levels of specialisation in producing different goods. Endogenous comparative advantage is much more important than exogenous comparative advantage since it is determined by individuals' decisions and level of division of labour. A general equilibrium model that can predict endogenous emergence of principal-agent relationship from evolution in division of labour can then be used to explore the implications of endogenous transaction costs for exploitation of endogenous comparative advantage. In most existing principal-agent models, exogenous comparative advantage which is based on *ex ante* differences between individuals is the driving force of the principal-agent relationship. The principal is *ex ante* different from the agent before decisions have been made. Hence, the existing literature has not explored the implications of endogenous transaction costs for exploitation of endogenous comparative advantage.

Endogenous transaction costs will affect in important ways the equilibrium pattern and level of division of labour which generate some concurrent phenomena (evolution in endogenous comparative advantage, in the degree of market integration, in the degree of diversity of economic structure, and so on) which cannot be predicted by standard marginal analysis of principal-agent models. Hence, the implications of endogenous transaction costs and contingent pricing for productivity progress and for all of the above economic phenomena can be explored using our general equilibrium model of principal-agent.

In our general equilibrium model, players' choices between contingent pricing and pure pricing is endogenised. Within a certain parameter subspace, individuals may prefer pure pricing to contingent pricing in general equilibrium. This feature of our general equilibrium model may be motivated by casual observations. We can see contingent pricing as well as pure pricing in real world under different conditions. The condition might not be as simple as the presence or absence of moral hazard. Our model will be used to identify the dividing line between contingent and pure pricing when moral hazard is present.

Section II specifies the model, Section III solves for corner equilibria in six market structures, Section IV solves for the general equilibrium and its comparative statics. In Section V, the implications of endogenous transaction costs for the equilibrium level of division of labour are explored. The final section concludes the paper.

## II. A General Equilibrium Model of Principal-Agent Relationship

In our model there are two *ex ante* identical consumer-producers. Each of them can produce a final good $y$ and an intermediate good $x$. Each person's utility function is specified as follows

$$U = \ln[(y + ky^d)s] \tag{1}$$

where $y$ is self-provided quantity of the final good, $y^d$ is the quantity of the final good purchased from the market. A fraction $1-k$ of a unit of goods purchased disappears in transit because of exogenous transaction costs. Hence, $ky^d$ is the quantity of the final good that is received from the purchase of $y^d$. $(y + ky^d)$ is thus the quantity of the final good that is consumed. $U$ is his utility level and $s$ is his level of leisure. This strictly concave utility function represents a preference with risk aversion.

Each person is equipped with the same production functions for the final and intermediate goods.

$$y^p \equiv y + y^s = (x + kx^d)L_y{}^4 \tag{2}$$

$$\text{if } L_x = \beta_H, \quad \text{then} \quad x^p \equiv x + x^s = \begin{cases} \theta_H & \text{with probability } \rho_H \\ \theta_L & \text{with probability } 1 - \rho_H \end{cases} \tag{3}$$

$$\text{if } L_x = \beta_L, \quad \text{then} \quad x^p \equiv x + x^s = \begin{cases} \theta_H & \text{with probability } \rho_L \\ \theta_L & \text{with probability} 1 - \rho_L \end{cases}$$

where $y^p$ and $x^p$ are respective output levels of the final and intermediate goods, $y^s$ and $x^s$ are respective quantities of the two goods sold, $x$ is self-provided quantity of the intermediate good, $x^d$ is the quantity of the intermediate good purchased, $L_y$ is the amount of labour allocated to produce good $y$. $L_x$ is effort level in reducing risk for a low productivity of $x$, which can be either a higher level $\beta_H$ or a lower level $\beta_L$. We call $L_i$ an individual's level of specialisation in producing good i.[5] $\beta_H$, $\beta_L$, $\theta_H$, $\theta_L$, $\rho_H$, and $\rho_L$ are parameters. The production function (2) indicates that total factor productivity of $y$, $y^p/(x+kx^d)^{1/2}L_y^{1/2}$, increases with an individual's level of specialisation in producing good $y$, $L_y$. This nature of production function is referred to as economies of specialisation. Here, the exponentially weighted average of two types of inputs, $(x+kx^d)^{1/2}L_y^{1/2}$, is total factor employed to produce $x$. Since economies of specialisation are individual specific and will not extend beyond the size of each person's limited amount of working time, they are localised increasing returns. A person's level of specialisation in producing a particular good increases as his range of activities is narrowed down. Hence, it is different from scale of his labour, despite the connection between the scale of labour and level of specialisation. The distinction between economies of specialisation and economies of scale is discussed in more details in Yang (1994). Each individual is endowed with $L$ units of time that can be allocated between working and leisure, so that the endowment constraint for each person is

$$L_x + L_y + s = L, \tag{4}$$

where $s$ is time allocated for leisure. This individual specific endowment constraint of time highlights the distinction between economies of specialisation and economies of scale. Finally, we assume

[4] If functional forms of the model are not explicitly specified, inframarginal comparative statics of general equilibrium cannot be solved because of complexity generated by many possible corner solutions. The result is not sensitive to the specification of functional forms as long as the trade-off between economies of specilisation and transaction costs exists. However, if Grossman's first order condition approach is adopted, results may be altered (see Yang, 2001, Ch. 9).

[5] $L_x$ represents a person's level of specilisation as well as his effort level in avoiding risk in producing $x$.

$$L_y \in [0, L], L_x = \beta_H \text{ or } \beta_L, s > 0, x, x^s, x^d, y, y^s, y^d \geq 0,$$
$$L > \beta_H > \beta_L > 0, \theta_H > \theta_L > 0,\ 1 > \rho_H > \rho_L > 0. \tag{5}$$

Each individual's self-interested behaviour is represented by a non-linear programming problem that maximises his expected utility with respect to $L_i$, $s$, $x$, $x^s$, $x^d$, $y$, $y^s$, $y^d$, subject to constraints (2)–(5).[6] When the decision variables $x$, $x^s$, $x^d$, $y$, $y^s$, $y^d$ take on 0 and positive values, there are $2^6 = 64$ combinations of 0 and positive values of the six variables. If two values of $L_x$ are considered too, there are 128 possible corner and interior solutions for each person's non-linear programming problem.

There are three periods in the model. Two players sign a contract on terms of trade in period 1. In period 2 they choose risk avoiding effort levels to maximise expected utility for given contractual terms. In period 3, Nature chooses a realized state and contractual terms are implemented. We assume that terms of trade are determined by a Nash bargaining game in period 1 when two players have free entry into the production of each good. Since two players are *ex ante* identical, it is easy to show that two players' expected utilities will be equalised in the Nash bargaining equilibrium. However, two players cannot change occupation in periods 2 and 3 after the contract is signed. The combination of moral hazard and endogenous specialisation generates a formidably large number of corner solutions. We have to reduce the number of corner solutions that we must consider to keep the model tractable.

Wen (1998) has shown that in these kinds of models, the interior solution is never optimal and that a person never simultaneously buys and sells the same good, never simultaneously buys and self-provides the same goods, never sells more than one good, and never self-provides the intermediate good if he sells that good. Following this theorem, we can rule out the interior solution and most corner solutions from consideration. A profile of 0 and positive values of decision variables that is consistent with the theorem is referred to as a configuration. Having considered possibilities for pure and contingent pricing for goods, we can identify 10 configurations that need to be considered. There is a corner solution for each configuration. A profile of matching configurations that is compatible with market clearing conditions is referred to as a market structure or simply a structure. There are 6 structures that need to be considered. The market clearing conditions, the incentive compatibility constraint, and utility equalisation conditions determine a corner equilibrium for each structure. The general equilibrium in the Nash bargaining game is a corner equilibrium in which nobody has incentive to deviate from his chosen configuration. We will adopt a two step approach to solving for the general equilibrium. First, all corner equilibria in the six structures are solved. Then the general equilibrium and its comparative statics will be identified.[7]

## III. Corner Equilibria in Six Structures

In this section, we specify, for each structure, a Nash bargaining game where two consumer-producers sort out terms of trade, sign contract, and then decide on their quantities of goods to consume, to produce, and to trade in period 1. They implement the contract after a state of $x^p$ is realized in period 2. In this regime, each person can observe outputs of the other

[6] Note that due to the budget constraint and market clearing condition, one of the decision variables is not independent of the others.

[7] This inframarginal approach is used by Dixit (1987, 1989a, b), Grossman and Hart (1986), and Hart and Moore (1990) too.

person, but cannot see his effort level $L_i$. The trade off between leisure and income from working is assumed, which together with non-verifiability of effort generates moral hazard. In the Nash bargaining game, each player, taking the relative price as given, maximises expected utility with respect to quantities of goods produced, traded, and consumed. The optimum decisions generate indirect utility functions for different occupation configurations. Then the Nash bargaining game will maximise the Nash product of the difference between two players' indirect utility functions and their utilities in autarky. Since two players are *ex ante* identical with the same autarky utility, the maximisation of the Nash product establishes utility equalisation between the two occupation configurations. Hence, the market clearing condition, utility equalisation, and incentive compatibility conditions determines a corner equilibrium for each structure. The Nash bargaining game will choose the structure with the highest expected utility. We first identify all structures that need to be considered. Then all corner equilibria in the structures will be solved.

(A) There are two autarchic structures $A_H$ and $A_L$, where there is no market and principal-agent relationship, and each individual self-provides all goods he consumes. Structure $A_H$ is composed of two individuals choosing configuration $A_H$, which implies a profile of decision variables with $L_x = \beta_H$, $L_y$; $x, y, s > 0$; $x^s = x^d = y^s = y^d = 0$. Structure $A_L$ is composed of two individuals choosing configuration $A_L$, which implies a profile of decision variables with $L_x = \beta_L$; $L_y, x, y, s > 0$; $x^s = x^d = y^s = y^d = 0$. The decision problem for a person choosing configuration $A_H$ is

$$\begin{aligned}
&\text{Max: } \mathrm{E}U = \ln y + \ln s && \text{(utility function)} \\
&\text{s.t. } y = xL_y && \text{(production function of } y) \\
&L_x = \beta_H, \quad x = \begin{cases} \theta_H & \text{with probability } \rho_H \\ \theta_L & \text{with probability } 1-\rho_H \end{cases} && \text{(production function of } x) \\
&L_x + L_y + s = L && \text{(endowment constraint)}
\end{aligned} \tag{6}$$

where E denotes expectation, $L_i$, $x, y, s$ are decision variables. The optimum solution for this problem and maximum expected utility for this configuration are listed in Table I. The decision problem for configuration $A_L$ can be obtained by replacing $L_x = \beta_H$ in problem (6) with $L_x = \beta_L$ and by replacing $\rho_H$ with $\rho_L$. Its optimum solution and expected maximum utility are listed in Table I.

(B) There are two market structures with the division of labour and unique relative price of the two goods, $D_L$ and $D_H$. Structure $D_L$ is comprised of a division of two individuals between configurations $(x/y)^u_L$ and $(y/x)^u_L$. Configuration $(x/y)^u_L$ implies that $L_x = \beta_L$;, s, x, $x^s$, $y^d > 0$; $L_y = x^d = y = y^s = 0$ and an individual choosing this configuration sells $x$, buys $y$, and accepts only a single relative price of the two goods. Configuration $(y/x)^u_L$ implies that $L_y, s, y, y^s, x^d > 0$, $L_x = y^d = x = x^s = 0$ and an individual choosing this configuration sells $y$, buys $x$, and accepts only a single relative price of the two goods.[8] Structure $D_H$ consists of a division of two individuals between configurations $(x/y)^u_H$ and $(y/x)^u_H$. Configuration $(x/y)^u_H$ is the same as $(x/y)^u_L$ except $L = \beta_H$. Configuration $(y/x)^u_H$ is the same as $(y/x)^u_L$ in structure $D_L$ except that value of $x^d$ will equal value of $x^s$ that is given by $L_x = \beta_H$ instead of by $L_x = \beta_L$.[9]

[8] If uncertainties are specified for the production of good $y$ and related configuration ($y/x$), then two sided moral hazard can be used to extend the Grossman-Hart-Moore model, as show in Yang (2000).
[9] As Hart (1995) indicated in a seminar, ownership of property does not make difference in the principal-agent model. The assumption that the specialist of $x$ or the specialist of $y$ owns $x$ and/or $y$ will not alter our results. For the implication of ownership when contracts are incomplete, see Grossman and Hart (1986) and Hart and Moore (1990).

**Table I** Corner solutions in 10 configurations

| *Configuration* | *Self-provided quantities* | *Demand and supply* | *Expected indirect utility* |
|---|---|---|---|
| $A_L$ | $x = \theta,\ y = 0.5(L - \beta_L)x$, $s = 0.5(L - \beta_L)$ | | $\rho_L \ln\theta_H + (1 - \rho_L)\ln\theta_L + 2\,[\ln(L - \beta_L) - \ln 2]$ |
| $A_H$ | $x = \theta,\ y = 0.5(L - \beta_H)x$, $s = 0.5(L - \beta_H)$ | | $\rho_H \ln\theta_H + (1 - \rho_H)\ln\theta_L + 2[\ln(L - \beta_H) - \ln 2]$ |
| $(x/y)_L{}^u$ | $s = L - \beta_L$ | $x^s = \theta\ y^d = px^s$ | $\ln p + \rho_L \ln\theta_H + (1 - \rho_L)\ln\theta_L + \ln(L - \beta_L) + \ln k$ |
| $(y/x)_L{}^u$ | $s = (kL - p)/2k$, $y = (kL - p)x^d/2$ | $x^d = x^s\ y^s = px^s$ | $2\ln(kL - p) + \rho_L \ln\theta_H + (1 - \rho_L)\ln\theta_L - 2\ln 2 - \ln k$ |
| $(x/y)_H{}^u$ | $s = L - \beta_H$ | $x^s = \theta\ y^d = px^s$ | $\ln p + \rho_H \ln\theta_H + (1 - \rho_H)\ln\theta_L + \ln(L - \beta_H) + \ln k$ |
| $(y/x)_H{}^u$ | $s = (kL + p)/2k$, $y = (kL + p)x^d/2$ | $x^d = x^s\ y^s = px^s$ | $2\ln(kL - p) + \rho_H \ln\theta_H + (1 - \rho_H)\ln\theta_L - 2\ln 2 - \ln k$ |
| $(x/y)_L{}^t$ | $s = L - \beta_L$ | $x^s = \theta\ y^d = \theta p$ | $\rho_L \ln(p_H\theta_H) + (1 - \rho_L)\ln(p_L\theta_L) + \ln(L - \beta_L) + \ln k$ |
| $(y/x)_L{}^t$ | $s = (kL - p)/2k$, $y = (kL - p)/2$ | $x^d = x^s\ y^s = p\theta$ | $\rho_L \ln[(kl_y - p_H)\theta_H] + (1 - \rho_L)\ln[(kl_y - p_L)\theta_L]$ |
| $(x/y)_H{}^t$ | $s = L - \beta_H$ | $x^s = \theta\ y^d = \theta p$ | $\rho_H \ln(p_H\theta_H) + (1 - \rho_H)\ln(p_L\theta_L) + \ln(L - \beta_H) + \ln k$ |
| $(y/x)_H{}^t$ | $s = (kL - p)/2k$, $y = (kL - p)/2$ | $x^d = x^s\ y^s = p$ | $\rho_H \ln[(kl_y - p_H)\theta_H] + (1 - \rho_H)\ln[(kl_y - p_L)\theta_L]$ |

Let us consider structure $D_H$ first. In this structure the decision problem for a person choosing configuration $(x/y)_H^u$ is

$$
\begin{aligned}
&\text{Max}: \mathrm{E}U_x = \ln(ky^d) + \ln s \\
&\text{s.t. } y^d = px^s && \text{(budget constraint)} \\
&L_x = \beta_H \quad x^s = \begin{cases} \theta_H & \text{with probability } \rho_H \\ \theta_L & \text{with probability } 1 - \rho_H \end{cases} && \text{(production function of } x) \\
&L_x + s = L && \text{(endowment constraint)}
\end{aligned}
\tag{7}
$$

where $L_x$, $x^s$, s are decision variables and $p \equiv p_x/p_y$ is the price of good $x$ in terms of good $y$ which is determined by Nash bargaining. Because of the market clearing condition $y^d = y^s$, he takes $y^s$ as given. The optimum decision for this configuration and expected indirect utility function are listed in Table I.

The decision problem for configuration $(y/x)^u$ is

$$
\begin{aligned}
&\text{Max}: \mathrm{E}U_y = \ln y + \ln s \\
&\text{s.t. } y^s = px^d && \text{(budget constraint)} \\
&y + y^s = (kx^d)L_y && \text{(production function of } y) \\
&L_y + s = L && \text{(endowment constraint)}
\end{aligned}
\tag{8}
$$

where $L_y$ and $s$ are decision variables and $p \equiv p_x/p_y$ is the price of good $x$ in terms of good $y$. Because of the market clearing conditions $x^d = x^s$ and the budget constraints for two configurations: $px^d = y^s$ and $y^d = px^s$, $x^d$ can be replaced with $x^s$, and $y^s$ is not independent of $x^s$. Hence, the person choosing this configuration takes $x^s$, which is chosen by a person in $(x/y)$, and $y^s = px^d = px^s$ as given, where $p$ is determined by Nash bargaining which equalises utility between the two configurations. The optimum decision for this configuration and expected indirect utility function are listed in Table I.

The utility equalisation condition gives the corner equilibrium solution of relative price $p$. Plugging the corner equilibrium relative price into utility function yields the corner equilibrium expected real income in structure $D_H$. All of the information on this corner equilibrium is in Table II. Replacing $L_x = \beta_H$ in structure $D_H$ with $L_x = \beta_L$and following the procedure for solving for the corner equilibrium in $D_H$, the corner equilibrium in structure $D_L$ can be solved. The corner equilibrium relative price and expected real income in $D_L$ are listed in Table II.

(C) There are two market structures with the division of labour and with two relative contingent prices, $C_L$ and $C_{H.}$ Structure $C_L$ consists of a division of two individuals between configurations $(x/y)_L^t$ and $(y/x)_L^t$. Configuration $(x/y)_L^t$ is the same as $(x/y)_L^u$ except that an individual choosing this configuration accepts relative price $p_H$ when output level of $x$ is high, or is $\theta_H$, and accepts relative price $p_L$ if output level of $x$ is $\theta_L$. Configuration $(y/x)_L^t$ is the same as $(y/x)_L^u$ except that an individual choosing this configuration accepts relative price $p_H$ when output level of $x$ is $\theta_H$, and accepts relative price $p_L$ if output level of $x$ is $\theta_L$. Structure $C_H$ is comprised of a division of two individuals between configurations $(x/y)_H^t$ and $(y/x)_H^t$ Configuration $(x/y)_H^t$ is the same as $(x/y)_L^t$ except that $L_x = \beta_L$ is replaced with $L_x = \beta_H$. Configuration $(y/x)_H^t$ is the same as in structure $C_L$ except that value of $x^d$ will equal value of $x^s$ that is given by $L_x = \beta_H$ instead of by $L_x = \beta_L$. The trade off between sharing risk and providing incentive is standard. Although the two players are *ex ante* identical, their occupations involve different *ex post* risk since configuration ($x/y$) involves uncertainty in production, while ($y/x$) does not. Hence, the specialist producer of $y$ has a trade off between providing the specialist producer of $x$ with strong incentive and sharing risk in producing $x$ with him via contingent terms of trade.

The procedure for solving for the corner equilibria in the two structures is the same as that for structures $D_L$ and $D_H$ except that the incentive compatibility condition is used, together with utility equalisation condition, to determine two corner equilibrium relative prices. Assume that the relative price is $p_H^i$ when output level of $x$ is $\theta_H$ and that it is $p_L^i$ when the output level is $\theta_L$, in structure $C_i$, where $i = L, H$. Let expected utility of the specialist choosing $(x/y)_H^t$, given in Table I, equal his expected utility for $(x/y)_L^t$, the incentive compatibility condition for $C_i$ can be derived as follows

$$\ln\left(\frac{p_L^i}{p_H^i}\right) = \ln\eta. \tag{9}$$

where $\eta$ is defined by $(\rho_H - \rho_L)\ln\eta \equiv (\rho_H - \rho_L)\ln\frac{\theta_H}{\theta_L} - \ln\frac{L-\beta_L}{L-\beta_H}$, and $p_H^i > p_L^i$ iff $\eta < 1$.

All information on the six corner equilibria is summarised in Table II.

**Table II** Corner equilibria in 6 structures

| *Structure* | *Corner equilibrium relative price* | *Expected real income* |
|---|---|---|
| $A_L$ | | $\rho_L\ln\theta_H + (1-\rho_L)\ln\theta_L + 2[\ln(L-\beta_L) - \ln 2]$ |
| $A_H$ | | $\rho_H\ln\theta_H + (1-\rho_H)\ln\theta_L + 2[\ln(L-\beta_H) - \ln 2]$ |
| $D_L$ | $p^L$ is given by (10) | $\ln p^L + \rho_L\ln\theta_H + (1-\rho_L)\ln\theta_L + \ln(L-\beta_L) + \ln k$ |
| $D_H$ | $p^H$ is given by (11) | $\ln p^H + \rho_H\ln\theta_H + (1-\rho_H)\ln\theta_L + \ln(L-\beta_H) + \ln k$ |
| $C_L$ | $p_L^H$ and $p_H^H$ are given by (9) and (12) | $\rho_L\ln(p_H^L\theta_H) + (1-\rho_L)\ln(p_L^L\,\theta_L) + \ln(L-\beta_L) + \ln k$ |
| $C_H$ | $p_L^H$ and $p_H^H$ are given by (9) and (13) | $\rho_H\ln(p_H^H\,\theta_H) + (1-\rho_H)\ln(p_L^H\,\theta_L) + \ln(L-\beta_H) + \ln k$ |

$$p^L = k\left\{[L + 2k(L - \beta_L)] - 2\sqrt{kL(L - \beta_L) + k^2(L - \beta_L)^2}\right\} \tag{10}$$

$$p^H = k\left\{[L + 2k(L - \beta_H)] - 2\sqrt{kL(L - \beta_H) + k^2(L - \beta_H)^2}\right\} \tag{11}$$

$$\begin{aligned} &\rho_L \ln \frac{p_H^L}{kl_y^L - p_H^L} + (1 - \rho_L) \ln \frac{p_L^L}{kl_y^L - p_L^L} + \ln \frac{k(L - \beta_L)}{L - l_y^L} = 0, \\ &\frac{k\rho_L}{kl_y^L - p_H^L} + \frac{k(1 - \rho_L)}{kl_y^L - p_L^L} = \frac{1}{L - l_y^L} \end{aligned} \tag{12}$$

$$\begin{aligned} &\rho_H \ln \frac{p_H^H}{kl_y^H - p_H^H} + (1 - \rho_H) \ln \frac{p_L^H}{kl_y^H - p_L^H} + \ln \frac{k(L - \beta_H)}{L - l_y^H} = 0, \\ &\frac{k\rho_H}{kl_y^H - p_H^H} + \frac{k(1 - \rho_H)}{kl_y^H - p_L^H} = \frac{1}{L - l_y^H} \end{aligned} \tag{13}$$

where $p_H^i$ is the price of good $x$ in terms of good $y$ when output level of $x$ is $\theta_H$ and $p_L^i$ is the relative price when output level of $x$ is $\theta_L$ in structure $C_i$. $p_H$ is the price of good $x$ in terms of good $y$ in structure $D_H$ and $p_L$ is that in structure $D_L$. In (12) or (13), $l_y^i = \phi \pm \sqrt{\gamma}$, where $\phi$ and $\gamma$ are dependent on $p_H^i, p_L^i, \rho, L, k$. We take $l_y^i = \phi + \sqrt{\gamma}$ to be the solution of $l_y^i$ since $kl_y^i - p_j^i > 0$ can be used to rule out the other solution.

The corner equilibrium in structure $D_H$ does not exist if the following condition holds

$$\eta < 1. \tag{14}$$

where $\eta$ is defined in (9). This condition implies that for a unique relative price of the two goods a specialist of $x$ will choose $L_x = \beta_L$, which is incompatible with the definition of structure $D_H$, if his expected utility is higher for $L_x = \beta_L$ than that for $L_x = \beta_H$. This is referred to as the problem of moral hazard. Similarly, the corner equilibrium in structure $D_L$ does not exist if $\eta > 1$. Hence, (14) gives ranges of parameter values that define the dividing line between the two corner equilibria. (14) implies that if benefit of effort in reducing risk of low productivity is outweighed by disutility of such effort, then the specialist of $x$ will choose the low effort level under unique relative price of the two goods.

## IV. General Equilibrium

In this section we first define general equilibrium, then analyse comparative statics and welfare implications of the general equilibrium.

General equilibrium is defined as a set of contingent or pure prices of goods and two players' labour allocations and trade plans that satisfy the following conditions. (i) Each individual's consumption plan generated by his labour allocation and trade plan maximises his expected utility for a given set of relative prices of traded goods; (ii) The set of relative prices of traded goods maximises the Nash product of the two players' differences of utilities between a chosen structure and autarky and clear the markets for traded goods subject to the incentive compatibility constraint. It is trivial to show that the Nash bargaining equilibrium is the corner equilibrium with the highest expected utility provided the incentive compatibility constraint is met. Therefore, solving for the general equilibrium becomes identifying the corner equilibrium with the highest expected real income subject to the incentive compatibility condition.[10] Comparisons of expected real incomes in 6 corner equilibria, given in Table II , and expressions (9)–(14) yield the following theorem.

*Theorem 1*

(1) For $\eta < \eta_0 < 1$
(1a) The general equilibrium is $A_L$ if $\beta_L < \frac{L}{5}$ , or if $\beta_L > \frac{L}{5}$ and $k < k_{1L}$ ;
(1b) The general equilibrium is $D_L$ if $\beta_L > \frac{L}{5}$ and $k > k_{1L}$.
where $k_{1L}$ is given by $4k^2\left(\frac{L}{L-\beta_L} + 2k - 2\sqrt{k\frac{L}{L-\beta_L} + k^2}\right) = 1$ (see lemma 7 in Appendix 1 for proof) and $\eta_0$ is given by $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) = 0$.
(2) For $\eta \in (\eta_0, 1)$
(2a) The general equilibrium is $A_L$ if $k < k_2$ and $\eta > \eta_1$ , or if $\eta < \eta_1$ ;
(2b) The general equilibrium is $C_H$ if $k > k_2$ and $\eta > \eta_1$;
where $k_2$ is given by (9), (13), and $\mathrm{E}U(C_H) = \mathrm{E}U(A_L)$, $\eta_1$ is given by $\mathrm{E}U(C_H) - \mathrm{E}U(A_L) = 0$ and $k = 1$.
(3) For $\eta \in (1, \eta_2)$ , where $\eta_2$ is defined by $\ln\eta_2 = \frac{1}{\rho_H-\rho_L}\ln\frac{L-\beta_L}{L-\beta_H}$.
(3a) The general equilibrium is $A_L$ if $k < k_3$ ;
(3b) The general equilibrium is $D_H$ if $k > k_3$ ;
where $k_3$ is given by $\ln[4kf(k, L, \beta_H)] - \ln\left(\left(\frac{L-\beta_H}{L-\beta_L}\right)^2 \Big/ \left(\frac{\theta_H}{\theta_L}\right)^{\rho_H-\rho_L}\right) = 0$ (see lemma 8 in Appendix 1 for proof).
(4) For $\eta > \eta_2$
(4a) The general equilibrium is $A_H$ if $\beta_H < \frac{L}{5}$ , or if $k < k_{1H}$ and $\beta_H > \frac{L}{5}$ ;
(4b) The general equilibrium is $D_H$ if $k > k_{1H}$ and $\beta_H > \frac{L}{5}$.

where $k_{1H}$ is given $4k^2\left(\frac{L}{L-\beta_H} + 2k - 2\sqrt{k\frac{L}{L-\beta_H} + k^2}\right) = 1$ (see (23) in lemma 7 in Appendix 1).

There are two types of comparative statics of general equilibrium of this model. Theorem 1 summarises the first type of them, referred to as inframarginal comparative statics. Inframarginal comparative statics imply that the general equilibrium will discontinuously jump across five of the six corner equilibria. The second type, referred to as marginal comparative statics, are conventional: endogenous variables continuously change in response to those changes in parameters that are within a parameter subspace demarked by inframarginal comparative statics. Theorem 1 has partitioned a 7 dimension parameter space into 8 subspaces and identifies which of 5 corner equilibria is general equilibrium within which parameter subspace. Solving for inframarginal comparative statics of general equilibrium is to solve for a system of simultaneous inequalities if the corner equilibrium expected per capita real income can be explicitly solved as functions of 7 parameters. But this is not easy for our model for three reasons. First, effects of 7 parameters $L, \beta_L, \beta_H, \rho_L, \rho_H, \theta_L, \theta_H$ on many endogenous variables must be considered. Second, when we analyse discontinuous jumps of the general equilibrium, 5 corner equilibria need to be considered, each of them is equivalent to a conventional general equilibrium. Finally, the

[10] The existence of equilibrium is not a problem in the explicitly specified model with nonlinear pricing so long as (10)–(13) have solutions.

corner equilibria in structure $C_i(i = L, H)$ cannot be solved analytically, but it is extremely difficult to identify the parameter subspace within which a particular structure generates the highest per capita real income in the absence of analytical expressions of the per capita real incomes in 6 corner equilibria. Hence, we need to prove 9 lemmas before proving theorem 1. Since the proof of the 9 lemmas and theorem 1 is quite cumbersome, we put it in the Appendix.

Table III summarises inframarginal comparative statics of general equilibrium, given by theorem 1. The bottom line in this table gives the equilibrium structure for each particular parameter subspace. In Table III , $A_L$ (or $A_H$) represents autarky with a low (or a high) effort level in avoiding low productivity risk, $D_L$ (or $D_H$) represents the structure with the division of labour, pure price, and a low (or a high) risk reducing effort level, $C_H$ represents the structure with the division of labour, contingent prices, and a high risk reducing effort level. Recalling the definition of $\eta$ in expression (9), we can see that $\eta$ represents benefit of risk reducing effort compared to its disutility. Bearing these definitions in mind, we can see from Table III that the dividing line between autarky and division of labour is the exogenous transaction efficiency coefficient $k$. If the transaction efficiency coefficient is small, the general equilibrium is autarky where no market and principal-agent relationship exists and productivity is low. If it is large, a structure with the division of labour and related reciprocal principal-agent relationships occurs in general equilibrium. If $\eta$ is very small ($\eta < \eta_0$), or if the benefit of risk reducing effort is insignificant compared to its cost, this effort level is always low in equilibrium, irrespective of the level of division of labour. If $\eta$ is very large ($\eta > \eta_2$), or if the benefit of risk reducing effort is great compared to its cost, this effort level is always high in equilibrium, irrespective of the level of division of labour.

If $\eta$ is neither very large nor very small ($\eta \in (\eta_0, \eta_2)$ , see columns 3–6 in Table III), then it is possible that as the exogenous transaction efficiency coefficient ($k$) increases from a low level to a high level, the general equilibrium jumps from autarky (column 3 or 5) where risk reducing effort level is low (structure $A_L$) to the division of labour where this effort level is high (structure $C_H$ in column 4 or $D_H$ in column 6 of Table III).

Having compared cases (1)–(4) in theorem 1 (or columns 1–8 in Table III), it can be seen that the emergence of the principal-agent relationship will involve two contingent relative prices if benefits of effort in reducing risk are neither too great nor too small compared to its disutility, that is, if $\eta$ is not too large nor too small. Otherwise, the emergence of the principal-agent relationship will involve a unique relative price of the two goods ($D_i$). An examination of (9) and Table III indicates that if structure $C_H$ is the general equilibrium, then $\eta < 1$ which implies $p_H^H > p_L^H$. Since $C_L$ is never a general equilibrium structure, this result implies that $p_H > p_L$ when two contingent relative prices take place in general equilibrium.

As we promised in the introductory section, our model can predict simultaneous increases in moral hazard, productivity, and equilibrium per capita real income within the parameter subspace $\eta \in (\eta_0, 1)$ which implies that productivity benefit of a high effort level in reducing risk is neither too great nor too small compared to its disutility. Within this parameter

**Table III** General equilibrium and its comparative statics

| $\eta < \eta_0$ | | $\eta \in (\eta_0, 1)$ | | $\eta \in (1, \eta_2)$ | | $\eta > \eta_2$ | |
|---|---|---|---|---|---|---|---|
| $\beta_L < L/5$ *or* $\beta_L > L/5$ *and* $k < k_{1L}$ | $\beta_L > L/5$ *and* $k > k_{1L}$ | $\eta < \eta_1$ *or* $\eta > \eta_1$ *and* $k < k_2$ | $\eta > \eta_1$ *and* $k > k_2$ | $k < k_3$ | $k > k_3$ | $\beta_H < L/5$ *or* $\beta_H > L/5$ *and* $k < k_{1H}$ | $\beta_H > L/5$ *and* $k > k_{1H}$ |
| $A_L$ | $D_L$ | $A_L$ | $C_H$ | $A_L$ | $D_H$ | $A_H$ | $D_H$ |

subspace, as transaction efficiency increases from a value smaller than $k_2$ to a value larger than $k_2$, the general equilibrium jumps from autarky with low effort level $A_L$ to the division of labour with high effort level and contingent pricing (structure $C_H$). This story is quite intuitive. If transaction efficiency is low and benefit of working hard is not that significant compared to its disutility, then individuals will choose autarky with low effort level. As transaction efficiency is sufficiently improved, individuals will choose the division of labour. The higher level of division of labour enlarges the scope for trading off productivity benefit of high effort level against its disutility since each individual now produces only one good and has more time to afford leisure. Also, the increase in transaction efficiency enlarges the scope for trading off positive network effect of division of labour against moral hazard, so that moral hazard, per capita real income, and productivity increase side by side as division of labour evolves. This predicts the phenomenon that a man works harder for others than for himself as the degree of commercialisation and related moral hazard increase.

The analysis of corner and general equilibrium can be summarised as follows. Each corner equilibrium sorts out resource allocation and contractual terms for a given level of division of labour and a given contractual regime. The general equilibrium sorts out the level of division of labour and contractual regime (pure price, $D_i$, or contingent prices, $C_i$, low or high level of effort in producing $x$) by efficiently trading off economies of specialisation against endogenous and exogenous transaction costs and by trading off benefits of a high level of effort against its disutility. The equilibrium level of division of labour is mainly determined by transaction efficiency compared to other parameters. The main determinant of contractual regime is parameter $\eta$ which represents benefits of a high level of effort compared to its disutility.

Figure 1 gives an intuitive illustration of exogenous evolution of division of labour based on the inframarginal comparative statics of the general equilibrium and related emergence of the reciprocal principal-agent relationships. Two individuals in panel (a) are used to illustrate autarky. Circles represent configurations. The lines represent flows of goods self-provided. In autarky there is no market demand and supply. Productivity is low because of a low level of specialisation. However, transaction costs do not exist because people do not do business with each other. Panel (b) represents the division of labour (structure D or C). The lines represent the flows of goods. For structure D or C, there are two markets, one for $x$ and the other for $y$. There are two distinctive professional sectors. The per capita output of each good is higher than in autarky because of a higher level of specialisation for each person. But transaction costs are higher than in autarky too because each person has to undertake two transactions to obtain the necessary consumption. The transactions incur exogenous as well as endogenous

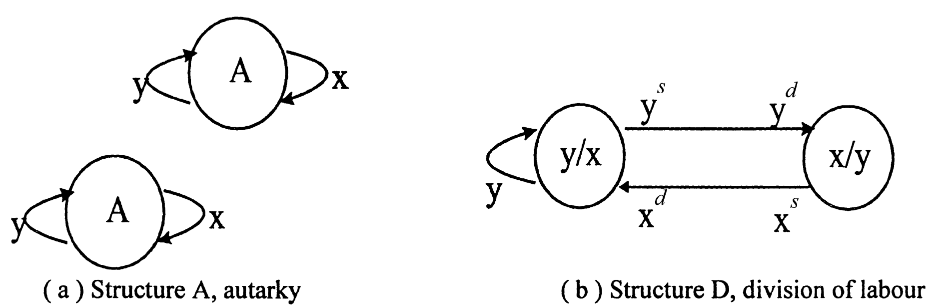


**Figure 1.** Evolution in division of labour and emergence of principal-agent relationships

transaction costs. Moreover, the degree of interdependence between individuals, the degree of production concentration of each good, the degree of diversity of economic structure, the degree of endogenous comparative advantage, and the degree of integration of society are higher in structure D or C than in autarky.

The implications of moral hazard for the equilibrium level of division of labour will be more significant if the number of goods is more than 2. The numbers of configurations and corner equilibria will increase more than proportionally as the numbers of goods increases in the model.

## V. Welfare Analysis and Endogenous Transaction Costs

We define a departure of equilibrium from the Pareto optimum as endogenous transaction cost and define exogenous transaction costs as a kind of transaction costs that can be identified before individuals have made decisions. Endogenous transaction cost cannot be identified before individuals have made decisions and the economy has settled down in equilibrium. In our model the transaction cost for a unit of goods purchased, $1-k$, is exogenous transaction cost.

Two features of the endogenous transaction costs in this model deserve special attention. A decrease in exogenous transaction cost $1-k$ (or an increase in $k$) may increase endogenous transaction costs and promote productivity as well as division of labour at the same time. For instance, in case (2) of theorem 1 (columns 3 and 4 of Table III), if $k < k_3$, then lemma 8 in the Appendix indicates that structure $A_L$ is the Pareto optimum. As $k$ increases, so that $k \in (k_3, k_2)$, the equilibrium is still $A_L$, but lemmas 2 and 8 in the Appendix indicate that $D_H$ is the Pareto optimum. Hence, the endogenous transaction costs keep the equilibrium structure and productivity from the efficient ones. This implies that an increase in $k$ creates endogenous transaction costs. As $k$ increases further, so that $k > k_2$. Then the equilibrium is $C_H$ and the Pareto optimum is $D_H$. This increase in $k$ generates endogenous transaction costs when it promotes division of labour and productivity, and increases each individual's utility. This predicts the phenomenon of simultaneous increases in moral hazard, productivity, and welfare. A member of a developed society with a greater degree of commercialisation (a higher level of division of labour) and more moral hazard is better off than in an autarchic less developed society with no much commercialisation and moral hazard.

However, in case (2b), structure $C_H$ may save on endogenous transaction costs if it is compared to $A_L$ and $D_H$. For this circumstance, the Pareto optimum that is associated with structure $D_H$ cannot occur in equilibrium due to moral hazard. If structure $C_H$ is not allowed, then the general equilibrium will be autarky with an even lower real income than in structure $D_H$. Hence, in this circumstance, structure $C_H$ maximises economies of division of labour net of endogenous and exogenous transaction costs despite the existence of endogenous transaction costs in structure $C_H$. This result substantiates the second part of the Coase theorem ( Coase, 1960) that the contractual structure in the market will maximise benefit of trade net of endogenous and exogenous transaction costs. It implies that it is not efficient to reduce endogenous transaction costs, which are considered as distortions, as much as possible because of the trade offs among three conflicting forces: economies of division of labour, endogenous transaction costs, and exogenous transaction costs. When benefits of effort in reducing risk are great, compared to its disutility, the conflict between a decrease in endogenous transaction costs and a decrease in exogenous transaction costs no longer exists, so that there is no trade off between economies of division of labour and endogenous transaction costs despite the existence of the trade off between economies of division of labour and exogenous transaction costs. In this sense, the model in this paper endogenises moral hazard and contingent prices. For cases in columns 5–8, the general equilibrium is Pareto optimal and endogenous transaction costs are not incurred.

The second of the features is that for the story in columns 3–6 of Table III, a sufficient improvement in transaction efficiency generates two kinds of gains from trade. It creates a greater scope for efficiently trading off economies of division of labour against endogenous and exogenous transaction costs, on the one hand, and creates greater scope for efficiently trading off benefits of effort in reducing risk against disutility of such effort. Hence, not only the equilibrium level of division of labour increases, but also the equilibrium level of effort in reducing risk increases. Both of such increases improve productivity.

As we claimed in the introductory section, the reciprocal principal-agent relationships may endogenously emerge from evolution in division of labour. In contrast, in the existing literature, the principal-agent relationship is exogenously given.[11]

## VI. Concluding Remarks

This paper has shown that the most important function of the price system is to efficiently trade off economies of division of labour against exogenous and endogenous transaction costs to sort out the equilibrium level of division of labour, the equilibrium extent of the market, and the equilibrium contractual arrangements that maximise positive network effect of the division of labour net of transaction costs. It is not efficient to reduce distortions (endogenous transaction costs) as much as possible because of the trade offs among economies of division of labour, endogenous transaction costs, and exogenous transaction costs. An inframarginal analysis is developed in this paper to sort out the inframarginal comparative statics of the general equilibrium that involve discontinuous jumps of general equilibrium between market structures as parameters shift between parameter subspaces. Each individual uses total benefit-cost analysis to choose his level and pattern of specialisation and contractual regime in addition to marginal analysis for sorting out resource allocation and contractual terms for a given pattern of specialisation and a given contractual regime. This model not only can predict emergence of reciprocal principal-agent relationships from evolution in division of labour, but also can endogenise comparative advantage and individuals' choice between a pure relative price and contingent relative prices. We use complicated trade offs between endogenous comparative advantage and endogenous and exogenous transaction costs to predict two interesting phenomena: (i) a man works harder for others in the presence of moral hazard than working for himself in the absence of moral hazard; (ii) simultaneous increases in welfare, productivity, and moral hazard. The two phenomena may occur if positive contribution of high effort level in reducing risk for low productivity is neither too great nor too small compared to its disutility and if transaction efficiency is improved.

[11] As Hart (1991) indicates, the principal-agent model in the present paper should not be considered as part of the formal theory of the firm since they have not endogenised asymmetric distribution of residual control rights and labour contract is not essential in the principal-agent model. The agent may claim part of residual returns from the contingent contract. The reciprocal principal-agent relationships in the model may exist in the absence of the institution of the firm, which is associated with asymmetric residual control and residual return and labour contract. Grossman and Hart (1986), Hart and Moore (1990), and Yang and Ng (1995) have developed formal models of institution of the firm.

A promising extension of our model is to follow Hart (1995) and Gupta and Romano (1998) to introduce two sided moral hazard and incomplete contracts into the model. The extended model may be able to explain the emergence of the institution of the firm from evolution in division of labour and to explore the implication of ownership structure for the equilibrium level of division of labour and related endogenous transaction costs.

## Appendix: Proof of Theorem 1

We first prove 9 lemmas that are essential for proving theorem 1.

*Lemma 1:* The corner equilibrium in $D_H$ cannot be a general equilibrium if $\ln \eta > 0$; the corner equilibrium in structure $D_L$ cannot be a general equilibrium if $\ln \eta > 0$.

*Proof.* The moral hazard indicated in (14) implies that for $\ln \eta > 0$ , specialists of $x$ have an incentive to choose $L_x = \beta_L$, so that the corner equilibrium in $D_H$ cannot be a general equilibrium. Similarly, for $\ln \eta > 0$, structure $D_L$ cannot be a general equilibrium.

*Lemma 2:* In the absence of moral hazard, the division of labour with a unique relative price of the two goods (structure $D_H$ or $D_L$) generates a greater expected real income than the corresponding structure with two contingent relative prices (structure $C_H$ or $C_L$).

*Proof.* From Table II, it can be shown that

$EU(C_i) < EU(D_i)$ iff

$$R_i \equiv \rho_H \ln p_H^i + (1 - \rho_H) \ln p_L^i < \ln p^i \tag{14}$$

where $i = L, H, p_H^i$ is the relative price when $x^s = \theta_H$ in structure $C_i, p_L^i$ is the relative price when $x^s = \theta_L$ in structure $C_i, p^i$ is the relative price in structure $D_i$. $p_H^L$ and $p_L^L$ are given as functions of all parameters by (9) and (12), and $p^L$ is given as a function of parameters by (10). $p_H^H$ and $p_L^H$ are given as functions of parameters by (9) and (13), and $p^H$ is given as a function of parameters by (11). $R_i$ is a function of $\eta$ which relates to parameters $\beta_i, \theta_i, \rho_i$. (9) and (13) gives that

$$p_H^i = p_L^i = p^i, \quad \text{when } \eta = 1 \tag{15}$$

where $p^i$ is the relative price in structure $D_i$ , given by (10) or (11). Hence, $p^i$ can be considered as a special solution of (9) and (13) for $\eta = 1$. (15) and the definition of $R_i$ in (14) together implies that

$$R(\eta = 1) = p^i \tag{16}$$

where the subscript of $R$ is removed since it does not make any difference when $\eta = 1$. Since $R_i$ in (14) is valued at any value of $\eta \neq 1$ the left hand side of the inequality in (14) can be redefined as a function of $\eta$ , or

$$R_i(\eta > 1) \text{ or } R_i(\eta < 1) \tag{17}$$

Replacing the left hand side of the inequality in (14) with (17) and the right hand side with (26) yields

$$R_i(\eta > 1) < R(\eta = 1) \text{ or } R_i(\eta < 1) < R(\eta = 1) \tag{18}$$

It is not difficult to see that (18) holds and therefore (14) holds if $dR_i/d\eta < 0$ for $\eta > 1$ and $dR_i/d\eta > 0$ for $\eta < 1$.

Using (9), the definition of $\eta$ in (9), and the first equation in (12) or in (13), we obtain an equation of $\eta, l_y^i$ and $p_H^i$. Total differentiating this equation with respect to $\eta$ and using the second equation in (12) or (13) yields

$$\frac{dR_i}{d\eta} < 0 \text{ for } \eta > 1 \quad \text{and} \quad \frac{dR_i}{d\eta} > 0 \text{ for } \eta < 1 \text{ if } \frac{dp_H^i}{d\eta} < 0 \tag{19}$$

Using (15) and differentiating the first equation in (12) or in (13) with respect to $\eta$ , then using the second equation in (12) or in (13) yields

$$\frac{dp_H^i}{d\eta} < 0 \tag{20}$$

(14), (18)–(20) are sufficient for establishing lemma 2.

*Remark:* Lemmas 1 and 2 imply that the corner equilibrium in $C_L$ cannot be a general equilibrium for $\ln\eta < 0$ and that $C_H$ is ruled out for $\ln\eta < 0$. (14) is analogous to the Jesen inequality, but it is much more difficult to prove than the Jesen inequality since $p^i$ is not necessarily a weighted average of $p_H^i$ and $p_L^i$.

*Lemma 3:* $\mathrm{E}U(C_L) < \mathrm{E}U(D_H)$ for $\eta > 1$.

*Proof.* From Table II , it can be shown

$$\mathrm{E}U(C_L) < \mathrm{E}U(D_H) \text{ iff } R_L(\eta) - \ln p^H - (\rho_H - \rho_L)\ln\eta < 0 \tag{21}$$

where $R_L(\eta)$ and $\eta$ are defined in (14) and (9), respectively. From lemma 2 $R_L(\eta) < \ln p^L$. From (15), $R_L(\eta) \to \ln p^L$ as $\eta \to 1$. According to (11) and (12), the corner equilibrium value of $p^i$ is a function of $\beta_i$. Denoting the function as $p^i(\beta_i)$ and differentiating (11) and (12) with respect to $\beta_i$ yields

$$dp^i(\beta_i)/d\beta_i > 0, \text{ so that } p^H(\beta_H) > p^L(\beta_L). \tag{22}$$

Therefore, $R_L(\eta) - \ln p^H - (\rho_H - \rho_L)\ln\eta < 0$ for $\eta > 1$. This establishes lemma 3.

*Lemma 4:* The corner equilibrium in $A_H$ cannot be a general equilibrium if $\ln\eta < 0$.

*Proof.* From Table II , it can be shown

$$\mathrm{E}U(A_H) > \mathrm{E}U(A_L) \text{ iff } \ln\eta > \frac{1}{\rho_H - \rho_L}\ln\frac{L - \beta_L}{L - \beta_H}$$

where $\frac{1}{\rho_H-\rho_L}$ ln $\frac{L-\beta_L}{L-\beta H} > 0$. This implies that the corner equilibrium in $A_H$ cannot be a general equilibrium if $\ln\eta < 0$.

*Lemma 5:* $\mathrm{E}U(D_H) > \mathrm{E}U(D_L)$ if $\mathrm{E}U(A_H) > \mathrm{E}U(A_L)$.

*Proof.* From Table II , it can be shown

$$\mathrm{E}U(D_H) - \mathrm{E}U(D_L) - [\mathrm{E}U(D_H) - \mathrm{E}U(D_L)] > 0 \text{ iff}$$
$$\left(\frac{L}{L-\beta_H} - \frac{L}{L-\beta_L}\right)\sqrt{k}\left(\sqrt{\frac{L}{L-\beta_H}+k} + \sqrt{\frac{L}{L-\beta_L}+k} - 2\sqrt{k}\right) > 0$$

The last inequality certainly holds because of assumption $\beta_H > \beta_L > 0$.

*Lemma 6:* $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) > 0$ as $\eta \to 1$ and $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) < 0$ as $\eta \to 0$. There exists $\eta_0 \in (0,1)$, such that $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) > 0$ iff $\eta > \eta_0$.

*Proof.* From Table II, it can be shown

$$\mathrm{E}U(C_H) > \mathrm{E}U(D_L) \text{ iff } R_H(\eta) - \ln p^L + (\rho_H - \rho_L)\ln\eta > 0$$

where $R_H(\eta)$ and $\eta$ are defined in (14) and (9), respectively. From lemma 2 $R_H(\eta) < \ln p^H$. From (15), $R_H(\eta) \to \ln p^H$ as $\eta \to 1$. From (28), $p^H > p^L$. Therefore,

$$R_H(\eta) - \ln p^L + (\rho_H - \rho_L)\ln\eta > 0 \text{ as } \eta \to 1.$$

Also, it can be seen from the definitions of $R_H$ and $\eta$ that

$$R_H(\eta) - \ln p^L + (\rho_H - \rho_L)\ln\eta = \ln\frac{p^H}{p^L} + (1-\rho_L)\ln\eta$$

where $(1-\rho_L)\ln\eta \to -\infty$ as $\eta \to 0$. From (11) and (12), it can be shown that $p^H/p^L$ tends to infinity only if $\beta_H - \beta_L$ tends to infinity. However, the assumption $\beta_H, \beta_L < L$ implies $\beta_H - \beta_L$ cannot be infinite. This means $p^H/p^L$ cannot be infinite. Therefore,

$$\mathrm{E}U(C_H) - \mathrm{E}U(D_L) = \ln\frac{p^H}{p^L} + (1-\rho_L)\ln\eta < 0 \text{ as } \quad \eta \to 0.$$

Hence, there exists $\eta_0 \in (0,1)$, such that $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) > 0$ iff $\eta > \eta_0$, where $\eta_0$ is given by $\mathrm{E}U(C_H) - \mathrm{E}U(D_L) = 0$. This establishes lemma 6.

*Lemma 7:* $\dfrac{d[\mathrm{E}U(D_i) - \mathrm{E}U(A_i)]}{dk} > 0$. $\mathrm{E}U(D_i) > \mathrm{E}U(A_i)$ iff $k > k_{1i}$, where $k_{1i}$ is given by

$$4kf(k, L, \beta_i) = 1, \tag{23a}$$

$$f(k, L, \beta_i) \equiv k\left(\frac{L}{L-\beta_i} + 2k - 2\sqrt{k\frac{L}{L-\beta_i} + k^2}\right) \tag{23b}$$

$k_{1i} \in (0,1)$ if $\beta_i > \frac{L}{5}$. $\mathrm{E}U(D_i) < \mathrm{E}U(A_i)$ if $k$ is sufficiently close to 0. Here $i = L, H$.

*Proof.* From Table II , it can be shown that

$$\frac{d[\mathrm{E}U(D_i) - \mathrm{E}U(A_i)]}{dk} > 1, \quad \text{iff} \frac{df(k, L, \beta_i)}{dk} > 0, \quad \text{iff} \left(\frac{L}{L - \beta_i}\right)^3 > 0. \tag{24}$$

where $i = L, H$. It is straightforward that the last inequality in (24) always holds because of the assumption $L > \beta_i$. Also, it can be shown that

$$\begin{aligned} &\mathrm{E}U(D_i) - \mathrm{E}U(A_i) \equiv \ln[4kf(k, L, \beta_i)] \to -\infty, \quad \text{as } k \to 0, \\ &\text{for } k = 1,\ U(D_i) - U(A_i) > 0 \quad \text{iff } \beta_i > \frac{L}{5} \end{aligned} \tag{25}$$

(24) and (25) are sufficient for establishing lemma 7.

*Lemma 8:* $\frac{d[\mathrm{E}U(D_H) - \mathrm{E}U(A_L)]}{dk} > 0$. $U(D_H) > U(A_L)$ if $k > k_3$, where $k_3$ is given by

$$\ln[4kf(k, L, \beta_H)] - \ln\left(\left(\frac{L - \beta_H}{L - \beta_L}\right)^2 \Big/ \left(\frac{\theta_H}{\theta_L}\right)^{\rho_H - \rho_L}\right) = 0.$$

and $k_3 \in (0,1)$. Here $f(k, L, \beta_i)$ is given in (23).

*Proof.* From Table II and expressions (11), it can be shown that

$$\frac{d[\mathrm{E}U(D_H) - \mathrm{E}U(A_L)]}{dk} > 0. \tag{26}$$

Also, it can be shown that

$$\mathrm{E}U(D_H) < \mathrm{E}U(A_L) \quad \text{if } k \to 0 \tag{27}$$

and when $k = 1, \mathrm{E}U(D_H) > \mathrm{E}U(A_L)$ iff $4\left(\frac{L}{L-\beta_H} + 2 - 2\sqrt{\frac{L}{L-\beta_H} + 1}\right)\left(\frac{\theta_H}{\theta_L}\right)^{\rho_H - \rho_L} > \left(\frac{L-\beta_L}{L-\beta_H}\right)^2$. It is not difficult to show that the last inequality always holds. This implies that

$$\text{when } k = 1,\ \text{we always have } U(D_H) > U(A_L). \tag{28}$$

(26)–(28) are sufficient for establishing lemma 8.

*Lemma 9.* $\frac{d[\mathrm{E}U(C_H) - \mathrm{E}U(A_L)]}{dk} > 0$. $\mathrm{E}U(C_H) > \mathrm{E}U(A_i)$ if $k > k_2$ and $\eta > \eta_1$. $\mathrm{E}U(C_H) < \mathrm{E}U(A_L)$ if $k < k_2$ and $\eta > \eta_1$ , or if $\eta < \eta_1$ , where $k_2$ is given by (9), (13) and $\mathrm{E}U(C_H) = \mathrm{E}U(A_L)$ and $\eta_1$ is given by $\mathrm{E}U(C_H) - \mathrm{E}U(A_L) = 0$ and $k = 1$.

*Proof.* Differentiating the first equation in (12) or in (13) with respect to $k$ yields

$$\frac{dp_j^i}{dk} > 0 \quad \text{iff } \frac{2l_y^i - L}{k(L - l_y^i)} > 0. \tag{29a}$$

where $i = L, H$ and $j = L, H$. It can be verified from the second equation in (12) or in (13) that

$$l_y^i > \frac{L}{2}. \tag{29b}$$

(29) implies that

$$\frac{dp_i^i}{dk} > 0 \tag{30}$$

Differentiating $\mathrm{E}U(C_H) - \mathrm{E}U(A_i)$ with respect to $k$ yields

$$d[\mathrm{E}U(C_H) - \mathrm{E}U(A_i)]/dk > 0 \quad \text{iff} \quad \frac{dp_i^H}{dk} > 0 \tag{31}$$

(30) and (31) implies

$$d[\mathrm{E}U(C_H) - \mathrm{E}U(A_i)]/dk > 0 \tag{32}$$

Also, it can be shown that

$$\text{For } k = 1, \mathrm{E}U(C_H) \to \mathrm{E}U(D_H) \quad \text{as } \eta \to 1 \tag{33}$$

$$\text{For } k = 1, \mathrm{E}U(C_H) < \mathrm{E}U(A_i) \quad \text{as } \eta \to 0 \tag{34a}$$

(33) and lemma 2 imply

$$\mathrm{E}U(C_H) < \mathrm{E}U(A_i) \quad \text{if } k \to 0, \tag{34b}$$

(28) and (33) imply

$$\mathrm{E}U(C_H) < \mathrm{E}U(A_i) \quad \text{for } k = 1 \text{ and } \eta \to 1. \tag{34c}$$

(34) implies that

There exists $\eta_1 \in (0, 1)$ such that when $k = 1$ $\mathrm{E}U(C_H) - \mathrm{E}U(A_L) > 0$ iff $\eta > \eta_1$, (35)

where $\eta_1$ is given by $\mathrm{E}U(C_H) - \mathrm{E}U(A_L) = 0$ and $k = 1$. (32) and (35) are sufficient for establishing lemma 9.

Now, we are ready to prove theorem 1.

*Proof of theorem 1* Lemma 1 implies that for $\eta < 1$, the set of candidates for general equilibrium consists of $A_L, A_H, D_L, C_L, C_H$. Lemmas 2 and 3 can be used to rule out $C_L$ from the set. Lemma 4 rules out $A_H$ from this set. Then lemmas 6, 7, and 9 are sufficient for establishing claims (1) and (2) in theorem 1.

Lemma 1 implies that for $\eta > 1$, the set of candidates for general equilibrium consists of $A_L, A_H, D_L, C_L, C_H$. Lemmas 2 rules out $C_H$ from the set. Now we consider case $\mathrm{E}U(A_H) > \mathrm{E}U(A_L)$ which holds iff $\eta > \eta_2$ and case $\mathrm{E}U(A_H) < \mathrm{E}U(A_L)$ which holds iff $\eta > \eta_2$, separately.

For case $1 < \eta < \eta_2$, lemma 3 can be used to rule out $C_L$ from the set of candidates for equilibrium. Then the set consists of only $A_L$ and $D_H$. Lemma 8 is thus sufficient for establishing claim (3) in theorem 1.

For case $\eta > \eta_2 > 1$, lemma 5 can be used to rule out $D_L$ from the set of candidates for equilibrium. Lemma 3 can be used to rule out $C_L$ from the set. Then the set consists of only $A_H$ and $D_H$. Lemma 7 is thus sufficient for establishing claim (4) in theorem 1.

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