



中央研究院數學研究所
Institute of Mathematics
Academia Sinica

國立台灣大學數學系
Department of Mathematics
National Taiwan University



Lakeside Lectures

Speaker: Professor Yoichi Miyaoka
(The University of Tokyo)

Title: Vector bundles associated
with Higgs bundles with
applications to the
Bogomolov inequality of
semistable Higgs bundles



Abstract:

A Higgs bundle on a smooth variety X is a pair (E, φ) of a vector bundle and a homomorphism $\varphi : E \rightarrow \Omega^1 \otimes E$. If a subsheaf F of E satisfies $\varphi(F) \subset \Omega^1 \otimes F$, it is called a Higgs subsheaf. When X is projective with an ample divisor H , we can define the semistability of a Higgs bundle (E, φ) in the same manner as in the case of usual vector bundles. In the lecture, we show that a Higgs bundle (E, φ) gives rise to a vector bundle \tilde{E} defined over a variety Y together with a surjective morphism $f: Y \rightarrow X$ such that (1) \tilde{E} has the same rank as E ; (2) Higgs subsheaves F of E corresponds to a subsheaf of \tilde{E} ; and (3) Semistability of E translates to a weak semistability of \tilde{E} . From the above correspondence between (E, φ) and \tilde{E} , we deduce several results such as: (1) Semistability is preserved by pull-backs and tensor products; (2) The 1st and the 2nd Chern classes of a semistable Higgs bundle satisfy the Bogomolov inequality. The Bogomolov inequality was first proved by Simpson by constructing harmonic metrics on Higgs bundles, which depends on hard analysis. Our approach gives a much more elementary alternative proof.

Date: Dec. 9th, 2013

Time: 14:00-15:00

Venue: Room 202, 2/F, Astro-Math. Building

Refreshment: 13:30-14:00

Organizers: Yi-Chiuan Chen, Chen-Yu Chi, Chun-Chung Hsieh, Jeng-Daw Yu